x, and the projectile is in place. (c) When released, the spring converts elastic potential energy $\mathrm{PE_{el}}$ into kinetic energy.

Strategy for a

(a): The energy stored in the spring can be found directly from elastic potential energy equation, because k and x are given.

Solution for a

Entering the given values for k and x yields

$$\begin{aligned}\mathrm{PE_{el}} &= \frac{1}{2}kx^2 = \frac{1}{2}(50.0\ \mathrm{N/m})(0.150\ \mathrm{m})^2 = 0.563\ \mathrm{N\cdot m} \\ &= 0.563\ \mathrm{J}\end{aligned} \tag{16.5}$$

Strategy for b

Because there is no friction, the potential energy is converted entirely into kinetic energy. The expression for kinetic energy can be solved for the projectile's speed.

Solution for b

1. Identify known quantities:

$$\mathrm{KE_f} = \mathrm{PE_{el}}\ \text{ or }\ 1/2\,mv^2 = (1/2)\,kx^2 = \mathrm{PE_{el}} = 0.563\ \mathrm{J} \tag{16.6}$$

2. Solve for v:

$$v = \left[\frac{2\mathrm{PE_{el}}}{m}\right]^{1/2} = \left[\frac{2(0.563\ \mathrm{J})}{0.002\ \mathrm{kg}}\right]^{1/2} = 23.7(\mathrm{J/kg})^{1/2} \tag{16.7}$$

3. Convert units: $23.7\ \mathrm{m/s}$

Discussion

(a) and (b): This projectile speed is impressive for a toy gun (more than 80 km/h). The numbers in this problem seem reasonable. The force needed to compress the spring is small enough for an adult to manage, and the energy imparted to the dart is small enough to limit the damage it might do, especially because the darts in many of these guns are made of soft material with a rubber tip. Yet, the speed of the dart is great enough for it to travel an acceptable distance.

CHECK YOUR UNDERSTANDING

Envision holding the end of a ruler with one hand and deforming it with the other. When you let go, you can see the oscillations of the ruler. In what way could you modify this simple experiment to increase the rigidity of the system?

Solution

You could hold the ruler at its midpoint so that the part of the ruler that oscillates is half as long as in the original experiment.

CHECK YOUR UNDERSTANDING

If you apply a deforming force on an object and let it come to equilibrium, what happened to the work you did on the system?

Solution

It was stored in the object as potential energy.

16.2 Period and Frequency in Oscillations

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Observe the vibrations of a guitar string.
- Determine the frequency of oscillations.

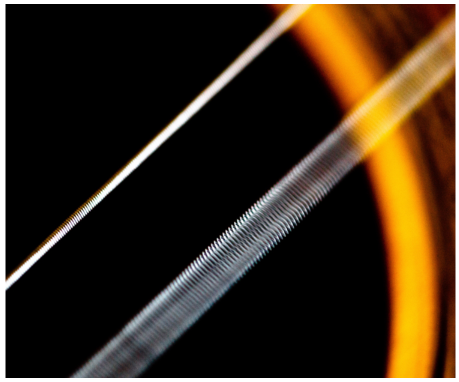

FIGURE 16.8 The strings on this guitar vibrate at regular time intervals. (credit: JAR)

When you pluck a guitar string, the resulting sound has a steady tone and lasts a long time. Each successive vibration of the string takes the same time as the previous one. We define **periodic motion** to be a motion that repeats itself at regular time intervals, such as exhibited by the guitar string or by an object on a spring moving up and down. The time to complete one oscillation remains constant and is called the **period** T. Its units are usually seconds, but may be any convenient unit of time. The word period refers to the time for some event whether repetitive or not; but we shall be primarily interested in periodic motion, which is by definition repetitive. A concept closely related to period is the frequency of an event. For example, if you get a paycheck twice a month, the frequency of payment is two per month and the period between checks is half a month. **Frequency** f is defined to be the number of events per unit time. For periodic motion, frequency is the number of oscillations per unit time. The relationship between frequency and period is

$$f = \frac{1}{T}. \tag{16.8}$$

The SI unit for frequency is the *cycle per second*, which is defined to be a *hertz* (Hz):

$$1\ \text{Hz} = 1\frac{\text{cycle}}{\text{s}} \text{ or } 1\ \text{Hz} = \frac{1}{\text{s}} \tag{16.9}$$

A cycle is one complete oscillation. Note that a vibration can be a single or multiple event, whereas oscillations are usually repetitive for a significant number of cycles.

EXAMPLE 16.3

Determine the Frequency of Two Oscillations: Medical Ultrasound and the Period of Middle C

We can use the formulas presented in this module to determine both the frequency based on known oscillations and the oscillation based on a known frequency. Let’s try one example of each. (a) A medical imaging device produces ultrasound by oscillating with a period of 0.400 µs. What is the frequency of this oscillation? (b) The frequency of middle C on a typical musical instrument is 264 Hz. What is the time for one complete oscillation?

Strategy

Both questions (a) and (b) can be answered using the relationship between period and frequency. In question (a), the period T is given and we are asked to find frequency f. In question (b), the frequency f is given and we are asked to find the period T.

Solution a

1. Substitute $0.400\ \mu\text{s}$ for T in $f = \frac{1}{T}$:

$$f = \frac{1}{T} = \frac{1}{0.400 \times 10^{-6}\ \text{s}}. \tag{16.10}$$

Solve to find

$$f = 2.50 \times 10^{6}\ \text{Hz}. \tag{16.11}$$

Discussion a

The frequency of sound found in (a) is much higher than the highest frequency that humans can hear and, therefore, is called ultrasound. Appropriate oscillations at this frequency generate ultrasound used for noninvasive medical diagnoses, such as observations of a fetus in the womb.

Solution b

1. Identify the known values:
   The time for one complete oscillation is the period T:

$$f = \frac{1}{T}. \tag{16.12}$$

2. Solve for T:

$$T = \frac{1}{f}. \tag{16.13}$$

3. Substitute the given value for the frequency into the resulting expression:

$$T = \frac{1}{f} = \frac{1}{264\text{ Hz}} = \frac{1}{264\text{ cycles/s}} = 3.79 \times 10^{-3}\text{ s} = 3.79\text{ ms}. \tag{16.14}$$

Discussion b

The period found in (b) is the time per cycle, but this value is often quoted as simply the time in convenient units (ms or milliseconds in this case).

CHECK YOUR UNDERSTANDING

Identify an event in your life (such as receiving a paycheck) that occurs regularly. Identify both the period and frequency of this event.

Solution

I visit my parents for dinner every other Sunday. The frequency of my visits is 26 per calendar year. The period is two weeks.

16.3 Simple Harmonic Motion: A Special Periodic Motion

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Describe a simple harmonic oscillator.
- Explain the link between simple harmonic motion and waves.

The oscillations of a system in which the net force can be described by Hooke's law are of special importance, because they are very common. They are also the simplest oscillatory systems. **Simple Harmonic Motion** (SHM) is the name given to oscillatory motion for a system where the net force can be described by Hooke's law, and such a system is called a **simple harmonic oscillator**. If the net force can be described by Hooke's law and there is no *damping* (by friction or other non-conservative forces), then a simple harmonic oscillator will oscillate with equal displacement on either side of the equilibrium position, as shown for an object on a spring in Figure 16.9. The maximum displacement from equilibrium is called the **amplitude** X. The units for amplitude and displacement are the same, but depend on the type of oscillation. For the object on the spring, the units of amplitude and displacement are meters; whereas for sound oscillations, they have units of pressure (and other types of oscillations have yet other units). Because amplitude is the maximum displacement, it is related to the energy in the oscillation.

Take-Home Experiment: SHM and the Marble

Find a bowl or basin that is shaped like a hemisphere on the inside. Place a marble inside the bowl and tilt the bowl periodically so the marble rolls from the bottom of the bowl to equally high points on the sides of the bowl.