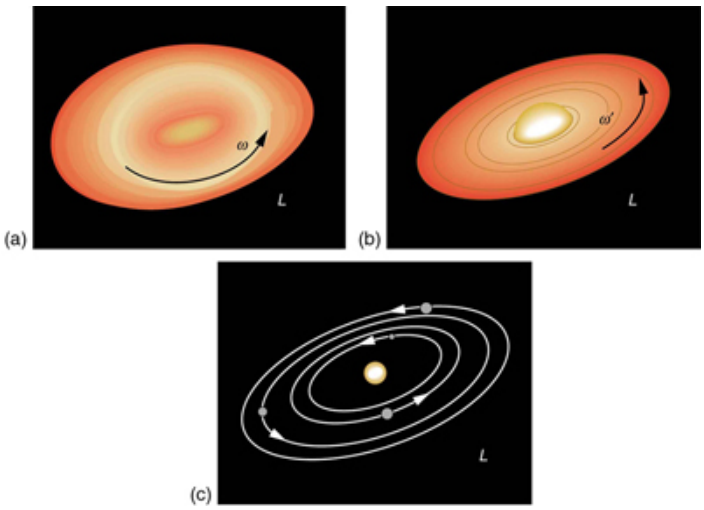


FIGURE 10.22 The Solar System coalesced from a cloud of gas and dust that was originally rotating. The orbital motions and spins of the planets are in the same direction as the original spin and conserve the angular momentum of the parent cloud.

In case of human motion, one would not expect angular momentum to be conserved when a body interacts with the environment as its foot pushes off the ground. Astronauts floating in space aboard the International Space Station have no angular momentum relative to the inside of the ship if they are motionless. Their bodies will continue to have this zero value no matter how they twist about as long as they do not give themselves a push off the side of the vessel.

### CHECK YOUR UNDERSTANDING

Is angular momentum completely analogous to linear momentum? What, if any, are their differences?

**Solution**

Yes, angular and linear momentums are completely analogous. While they are exact analogs they have different units and are not directly inter-convertible like forms of energy are.

## 10.6 Collisions of Extended Bodies in Two Dimensions

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Observe collisions of extended bodies in two dimensions.
- Examine collision at the point of percussion.

Bowling pins are sent flying and spinning when hit by a bowling ball—angular momentum as well as linear momentum and energy have been imparted to the pins. (See Figure 10.23). Many collisions involve angular momentum. Cars, for example, may spin and collide on ice or a wet surface. Baseball pitchers throw curves by putting spin on the baseball. A tennis player can put a lot of top spin on the tennis ball which causes it to dive down onto the court once it crosses the net. We now take a brief look at what happens when objects that can rotate collide.

Consider the relatively simple collision shown in Figure 10.24, in which a disk strikes and adheres to an initially motionless stick nailed at one end to a frictionless surface. After the collision, the two rotate about the nail. There is an unbalanced external force on the system at the nail. This force exerts no torque because its lever arm $r$ is zero. Angular momentum is therefore conserved in the collision. Kinetic energy is not conserved, because the collision is inelastic. It is possible that momentum is not conserved either because the force at the nail may have a component in the direction of the disk's initial velocity. Let us examine a case of rotation in a collision in Example 10.15.

FIGURE 10.23 The bowling ball causes the pins to fly, some of them spinning violently. (credit: Tinou Bao, Flickr)

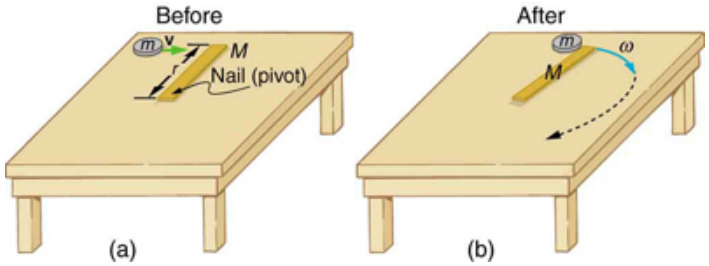


FIGURE 10.24 (a) A disk slides toward a motionless stick on a frictionless surface. (b) The disk hits the stick at one end and adheres to it, and they rotate together, pivoting around the nail. Angular momentum is conserved for this inelastic collision because the surface is frictionless and the unbalanced external force at the nail exerts no torque.

## EXAMPLE 10.15

### Rotation in a Collision

Suppose the disk in Figure 10.24 has a mass of 50.0 g and an initial velocity of 30.0 m/s when it strikes the stick that is 1.20 m long and 2.00 kg.

(a) What is the angular velocity of the two after the collision?

(b) What is the kinetic energy before and after the collision?

(c) What is the total linear momentum before and after the collision?

**Strategy for (a)**

We can answer the first question using conservation of angular momentum as noted. Because angular momentum is $I\omega$, we can solve for angular velocity.

**Solution for (a)**

Conservation of angular momentum states

$$L = L', \tag{10.122}$$

where primed quantities stand for conditions after the collision and both momenta are calculated relative to the pivot point. The initial angular momentum of the system of stick-disk is that of the disk just before it strikes the stick. That is,

$$L = I\omega, \tag{10.123}$$

where $I$ is the moment of inertia of the disk and $\omega$ is its angular velocity around the pivot point. Now, $I = mr^2$ (taking the disk to be approximately a point mass) and $\omega = v/r$, so that

$$L = mr^2\frac{v}{r} = mvr. \tag{10.124}$$

After the collision,

$$L' = I'\omega'. \tag{10.125}$$

It is $\omega'$ that we wish to find. Conservation of angular momentum gives

$$I'\omega' = mvr. \tag{10.126}$$

Rearranging the equation yields

$$\omega' = \frac{mvr}{I'}, \tag{10.127}$$

where $I'$ is the moment of inertia of the stick and disk stuck together, which is the sum of their individual moments of inertia about the nail. Figure 10.11 gives the formula for a rod rotating around one end to be $I = Mr^2/3$. Thus,

$$I' = mr^2 + \frac{Mr^2}{3} = \left(m + \frac{M}{3}\right)r^2. \tag{10.128}$$

Entering known values in this equation yields,

$$I' = (0.0500 \text{ kg} + 0.667 \text{ kg})(1.20 \text{ m})^2 = 1.032 \text{ kg} \cdot \text{m}^2. \tag{10.129}$$

The value of $I'$ is now entered into the expression for $\omega'$, which yields

$$\begin{aligned} \omega' &= \frac{mvr}{I'} = \frac{(0.0500 \text{ kg})(30.0 \text{ m/s})(1.20 \text{ m})}{1.032 \text{ kg}\cdot\text{m}^2} \\ &= 1.744 \text{ rad/s} \approx 1.74 \text{ rad/s}. \end{aligned} \tag{10.130}$$

**Strategy for (b)**

The kinetic energy before the collision is the incoming disk's translational kinetic energy, and after the collision, it is the rotational kinetic energy of the two stuck together.

**Solution for (b)**

First, we calculate the translational kinetic energy by entering given values for the mass and speed of the incoming disk.

$$\text{KE} = \frac{1}{2}mv^2 = (0.500)(0.0500 \text{ kg})(30.0 \text{ m/s})^2 = 22.5 \text{ J} \tag{10.131}$$

After the collision, the rotational kinetic energy can be found because we now know the final angular velocity and the final moment of inertia. Thus, entering the values into the rotational kinetic energy equation gives

$$\begin{aligned} \text{KE}' &= \tfrac{1}{2}I'\omega'^2 = (0.5)\left(1.032 \text{ kg} \cdot \text{m}^2\right)\left(1.744 \tfrac{\text{rad}}{\text{s}}\right)^2 \\ &= 1.57 \text{ J}. \end{aligned} \tag{10.132}$$

**Strategy for (c)**

The linear momentum before the collision is that of the disk. After the collision, it is the sum of the disk's momentum and that of the center of mass of the stick.

**Solution of (c)**

Before the collision, then, linear momentum is

$$p = mv = (0.0500 \text{ kg})(30.0 \text{ m/s}) = 1.50 \text{ kg} \cdot \text{m/s}. \tag{10.133}$$

After the collision, the disk and the stick's center of mass move in the same direction. The total linear momentum is that of the disk moving at a new velocity $v' = r\omega'$ plus that of the stick's center of mass,

which moves at half this speed because $v_{\text{CM}} = \left(\frac{r}{2}\right)\omega' = \frac{v'}{2}$. Thus,

$$p' = mv' + Mv_{\text{CM}} = mv' + \frac{Mv'}{2}. \tag{10.134}$$

Gathering similar terms in the equation yields,

$$p' = \left(m + \frac{M}{2}\right)v' \tag{10.135}$$

so that

$$p' = \left(m + \frac{M}{2}\right)r\omega'. \tag{10.136}$$

Substituting known values into the equation,

$$p' = (1.050\ \text{kg})(1.20\ \text{m})(1.744\ \text{rad/s}) = 2.20\ \text{kg} \cdot \text{m/s}. \tag{10.137}$$

**Discussion**

First note that the kinetic energy is less after the collision, as predicted, because the collision is inelastic. More surprising is that the momentum after the collision is actually greater than before the collision. This result can be understood if you consider how the nail affects the stick and vice versa. Apparently, the stick pushes backward on the nail when first struck by the disk. The nail's reaction (consistent with Newton's third law) is to push forward on the stick, imparting momentum to it in the same direction in which the disk was initially moving, thereby increasing the momentum of the system.

The above example has other implications. For example, what would happen if the disk hit very close to the nail? Obviously, a force would be exerted on the nail in the forward direction. So, when the stick is struck at the end farthest from the nail, a backward force is exerted on the nail, and when it is hit at the end nearest the nail, a forward force is exerted on the nail. Thus, striking it at a certain point in between produces no force on the nail. This intermediate point is known as the *percussion point*.

An analogous situation occurs in tennis as seen in Figure 10.25. If you hit a ball with the end of your racquet, the handle is pulled away from your hand. If you hit a ball much farther down, for example, on the shaft of the racquet, the handle is pushed into your palm. And if you hit the ball at the racquet's percussion point (what some people call the "sweet spot"), then little or *no* force is exerted on your hand, and there is less vibration, reducing chances of a tennis elbow. The same effect occurs for a baseball bat.

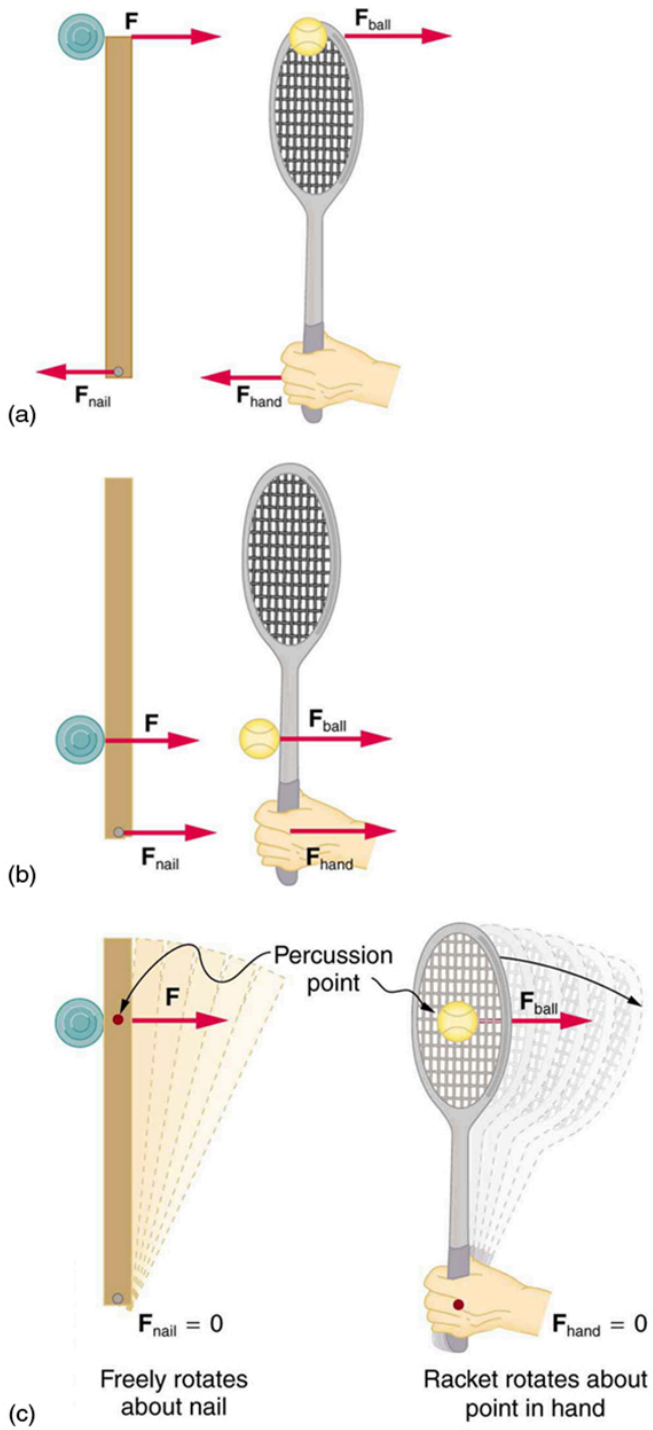


FIGURE 10.25 A disk hitting a stick is compared to a tennis ball being hit by a racquet. (a) When the ball strikes the racquet near the end, a backward force is exerted on the hand. (b) When the racquet is struck much farther down, a forward force is exerted on the hand. (c) When the racquet is struck at the percussion point, no force is delivered to the hand.

## CHECK YOUR UNDERSTANDING

Is rotational kinetic energy a vector? Justify your answer.

### Solution

No, energy is always scalar whether motion is involved or not. No form of energy has a direction in space and you can see that rotational kinetic energy does not depend on the direction of motion just as linear kinetic energy is independent of the direction of motion.