

3.1 The Laws of Planetary Motion

Learning Objectives

By the end of this section, you will be able to:

- Describe how Tycho Brahe and Johannes Kepler contributed to our understanding of how planets move around the Sun
- Explain Kepler's three laws of planetary motion

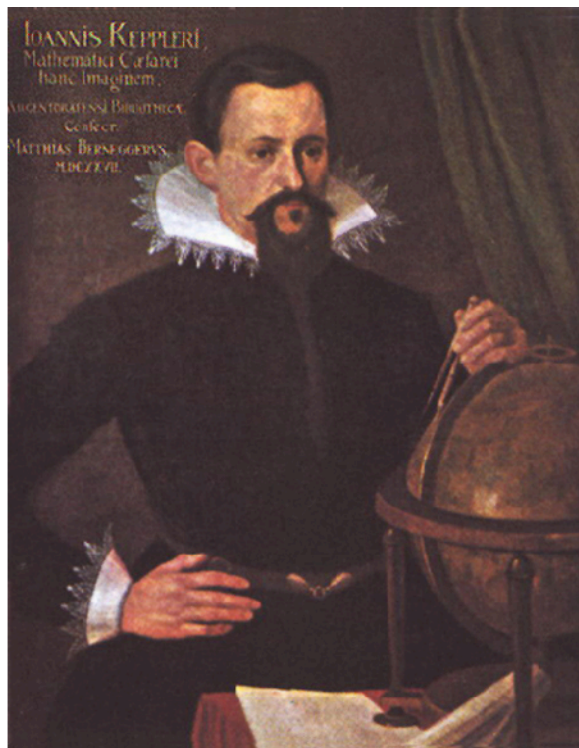
At about the time that Galileo was beginning his experiments with falling bodies, the efforts of two other scientists dramatically advanced our understanding of the motions of the planets. These two astronomers were the observer Tycho Brahe and the mathematician Johannes Kepler. Together, they placed the speculations of Copernicus on a sound mathematical basis and paved the way for the work of Isaac Newton in the next century.

Tycho Brahe's Observatory

Three years after the publication of Copernicus' *De Revolutionibus*, Tycho Brahe was born to a family of Danish nobility. He developed an early interest in astronomy and, as a young man, made significant astronomical observations. Among these was a careful study of what we now know was an exploding star that flared up to great brilliance in the night sky. His growing reputation gained him the patronage of the Danish King Frederick II, and at the age of 30, Brahe was able to establish a fine astronomical observatory on the North Sea island of Hven (Figure 3.2). Brahe was the last and greatest of the pre-telescopic observers in Europe.



(a)



(b)

Figure 3.2 Tycho Brahe (1546–1601) and Johannes Kepler (1571–1630). (a) A stylized engraving shows Tycho Brahe using his instruments to measure the altitude of celestial objects above the horizon. The large curved instrument in the foreground allowed him to measure precise angles in the sky. Note that the scene includes hints of the grandeur of Brahe's observatory at Hven. (b) Kepler was a German mathematician and astronomer. His discovery of the basic laws that describe planetary motion placed the heliocentric cosmology of Copernicus on a firm mathematical basis.

At Hven, Brahe made a continuous record of the positions of the Sun, Moon, and planets for almost 20 years. His extensive and precise observations enabled him to note that the positions of the planets varied from those

given in published tables, which were based on the work of Ptolemy. These data were extremely valuable, but Brahe didn't have the ability to analyze them and develop a better model than what Ptolemy had published. He was further inhibited because he was an extravagant and cantankerous fellow, and he accumulated enemies among government officials. When his patron, Frederick II, died in 1597, Brahe lost his political base and decided to leave Denmark. He took up residence in Prague, where he became court astronomer to Emperor Rudolf of Bohemia. There, in the year before his death, Brahe found a most able young mathematician, Johannes Kepler, to assist him in analyzing his extensive planetary data.

Johannes Kepler

Johannes Kepler was born into a poor family in the German province of Württemberg and lived much of his life amid the turmoil of the Thirty Years' War (see [Figure 3.2](#)). He attended university at Tübingen and studied for a theological career. There, he learned the principles of the Copernican system and became converted to the heliocentric hypothesis. Eventually, Kepler went to Prague to serve as an assistant to Brahe, who set him to work trying to find a satisfactory theory of planetary motion—one that was compatible with the long series of observations made at Hven. Brahe was reluctant to provide Kepler with much material at any one time for fear that Kepler would discover the secrets of the universal motion by himself, thereby robbing Brahe of some of the glory. Only after Brahe's death in 1601 did Kepler get full possession of the priceless records. Their study occupied most of Kepler's time for more than 20 years.

Through his analysis of the motions of the planets, Kepler developed a series of principles, now known as *Kepler's three laws*, which described the behavior of planets based on their paths through space. The first two laws of planetary motion were published in 1609 in *The New Astronomy*. Their discovery was a profound step in the development of modern science.

The First Two Laws of Planetary Motion

The path of an object through space is called its **orbit**. Kepler initially assumed that the orbits of planets were circles, but doing so did not allow him to find orbits that were consistent with Brahe's observations. Working with the data for Mars, he eventually discovered that the orbit of that planet had the shape of a somewhat flattened circle, or **ellipse**. Next to the circle, the ellipse is the simplest kind of closed curve, belonging to a family of curves known as *conic sections* ([Figure 3.3](#)).

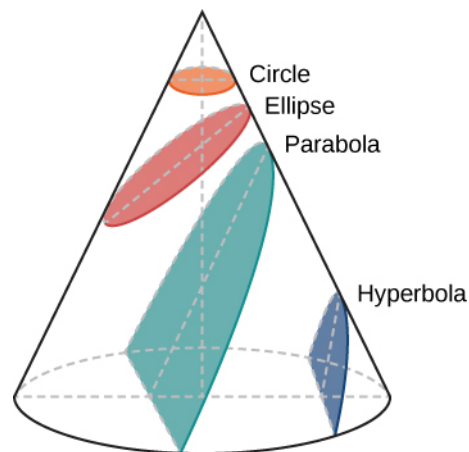


Figure 3.3 Conic Sections. The circle, ellipse, parabola, and hyperbola are all formed by the intersection of a plane with a cone. This is why such curves are called conic sections.

You might recall from math classes that in a circle, the center is a special point. The distance from the center to anywhere on the circle is exactly the same. In an ellipse, the sum of the distance from two special points inside the ellipse to any point on the ellipse is always the same. These two points inside the ellipse are called its foci (singular: **focus**), a word invented for this purpose by Kepler.

This property suggests a simple way to draw an ellipse ([Figure 3.4](#)). We wrap the ends of a loop of string

around two tacks pushed through a sheet of paper into a drawing board, so that the string is slack. If we push a pencil against the string, making the string taut, and then slide the pencil against the string all around the tacks, the curve that results is an ellipse. At any point where the pencil may be, the sum of the distances from the pencil to the two tacks is a constant length—the length of the string. The tacks are at the two foci of the ellipse.

The widest diameter of the ellipse is called its **major axis**. Half this distance—that is, the distance from the center of the ellipse to one end—is the **semimajor axis**, which is usually used to specify the size of the ellipse. For example, the semimajor axis of the orbit of Mars, which is also the planet's average distance from the Sun, is 228 million kilometers.

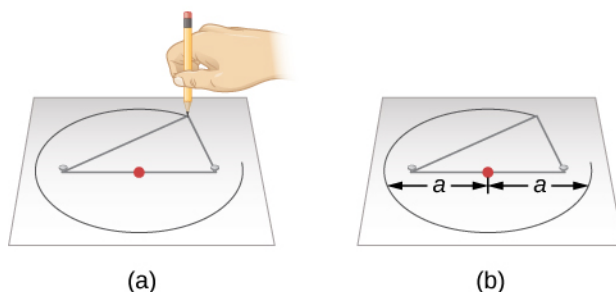


Figure 3.4 Drawing an Ellipse. (a) We can construct an ellipse by pushing two tacks (the white objects) into a piece of paper on a drawing board, and then looping a string around the tacks. Each tack represents a focus of the ellipse, with one of the tacks being the Sun. Stretch the string tight using a pencil, and then move the pencil around the tacks. The length of the string remains the same, so that the sum of the distances from any point on the ellipse to the foci is always constant. (b) In this illustration, each semimajor axis is denoted by a . The distance $2a$ is called the major axis of the ellipse.

The shape (roundness) of an ellipse depends on how close together the two foci are, compared with the major axis. The ratio of the distance between the foci to the length of the major axis is called the **eccentricity** of the ellipse.

If the foci (or tacks) are moved to the same location, then the distance between the foci would be zero. This means that the eccentricity is zero and the ellipse is just a circle; thus, a circle can be called an ellipse of zero eccentricity. In a circle, the semimajor axis would be the radius.

Next, we can make ellipses of various elongations (or extended lengths) by varying the spacing of the tacks (as long as they are not farther apart than the length of the string). The greater the eccentricity, the more elongated is the ellipse, up to a maximum eccentricity of 1.0, when the ellipse becomes “flat,” the other extreme from a circle.

The size and shape of an ellipse are completely specified by its semimajor axis and its eccentricity. Using Brahe's data, Kepler found that Mars has an elliptical orbit, with the Sun at one focus (the other focus is empty). The eccentricity of the orbit of Mars is only about 0.1; its orbit, drawn to scale, would be practically indistinguishable from a circle, but the difference turned out to be critical for understanding planetary motions.

Kepler generalized this result in his first law and said that *the orbits of all the planets are ellipses*. Here was a decisive moment in the history of human thought: it was not necessary to have only circles in order to have an acceptable cosmos. The universe could be a bit more complex than the Greek philosophers had wanted it to be.

Kepler's second law deals with the speed with which each planet moves along its ellipse, also known as its **orbital speed**. Working with Brahe's observations of Mars, Kepler discovered that the planet speeds up as it comes closer to the Sun and slows down as it pulls away from the Sun. He expressed the precise form of this relationship by imagining that the Sun and Mars are connected by a straight, elastic line. When Mars is closer to the Sun (positions 1 and 2 in [Figure 3.5](#)), the elastic line is not stretched as much, and the planet moves rapidly. Farther from the Sun, as in positions 3 and 4, the line is stretched a lot, and the planet does not move

so fast. As Mars travels in its elliptical orbit around the Sun, the elastic line sweeps out areas of the ellipse as it moves (the colored regions in our figure). Kepler found that in equal intervals of time (t), the areas swept out in space by this imaginary line are always equal; that is, the area of the region B from 1 to 2 is the same as that of region A from 3 to 4.

If a planet moves in a circular orbit, the elastic line is always stretched the same amount and the planet moves at a constant speed around its orbit. But, as Kepler discovered, in most orbits that speed of a planet orbiting its star (or moon orbiting its planet) tends to vary because the orbit is elliptical.

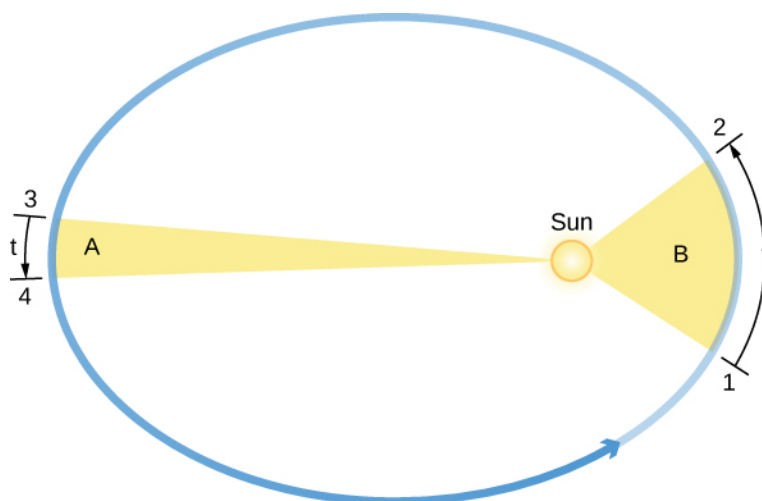


Figure 3.5 Kepler's Second Law: The Law of Equal Areas. The orbital speed of a planet traveling around the Sun (the circular object inside the ellipse) varies in such a way that in equal intervals of time (t), a line between the Sun and a planet sweeps out equal areas (A and B). Note that the eccentricities of the planets' orbits in our solar system are substantially less than shown here.

LINK TO LEARNING



The [Kepler's Second Law demonstrator \(https://openstax.org/l/30kepsecond\)](https://openstax.org/l/30kepsecond) from CCNY's ScienceSims project shows how an orbiting planet sweeps out the same area in the same time.

Kepler's Third Law

Kepler's first two laws of planetary motion describe the shape of a planet's orbit and allow us to calculate the speed of its motion at any point in the orbit. Kepler was pleased to have discovered such fundamental rules, but they did not satisfy his quest to fully understand planetary motions. He wanted to know why the orbits of the planets were spaced as they are and to find a mathematical pattern in their movements—a "harmony of the spheres" as he called it. For many years he worked to discover mathematical relationships governing planetary spacing and the time each planet took to go around the Sun.

In 1619, Kepler discovered a basic relationship to relate the planets' orbits to their relative distances from the Sun. We define a planet's **orbital period**, (P), as the time it takes a planet to travel once around the Sun. Also, recall that a planet's **semimajor axis**, a , is equal to its average distance from the Sun. The relationship, now known as *Kepler's third law*, says that a planet's orbital period squared is proportional to the semimajor axis of its orbit cubed, or

$$P^2 \propto a^3$$

When P (the orbital period) is measured in years, and a is expressed in a quantity known as an **astronomical unit (AU)**, the two sides of the formula are not only proportional but equal. One AU is the average distance between Earth and the Sun and is approximately equal to 1.5×10^8 kilometers. In these units,

$$P^2 = a^3$$

Kepler's third law applies to all objects orbiting the Sun, including Earth, and provides a means for calculating their relative distances from the Sun from the time they take to orbit. Let's look at a specific example to illustrate how useful Kepler's third law is.

For instance, suppose you time how long Mars takes to go around the Sun (in Earth years). Kepler's third law can then be used to calculate Mars' average distance from the Sun. Mars' orbital period (1.88 Earth years) squared, or P^2 , is $1.88^2 = 3.53$, and according to the equation for Kepler's third law, this equals the cube of its semimajor axis, or a^3 . So what number must be cubed to give 3.53? The answer is 1.52 (since $1.52 \times 1.52 \times 1.52 = 3.53$). Thus, Mars' semimajor axis in astronomical units must be 1.52 AU. In other words, to go around the Sun in a little less than two years, Mars must be about 50% (half again) as far from the Sun as Earth is.

EXAMPLE 3.1

Calculating Periods

Imagine an object is traveling around the Sun. What would be the orbital period of the object if its orbit has a semimajor axis of 50 AU?

Solution

From Kepler's third law, we know that (when we use units of years and AU)

$$P^2 = a^3$$

If the object's orbit has a semimajor axis of 50 AU ($a = 50$), we can cube 50 and then take the square root of the result to get P:

$$P = \sqrt{a^3}$$

$$P = \sqrt{50 \times 50 \times 50} = \sqrt{125,000} = 353.6 \text{ years}$$

Therefore, the orbital period of the object is about 350 years. This would place our hypothetical object beyond the orbit of Pluto.

Check Your Learning

What would be the orbital period of an asteroid (a rocky chunk between Mars and Jupiter) with a semimajor axis of 3 AU?

Answer:

$$P = \sqrt{3 \times 3 \times 3} = \sqrt{27} = 5.2 \text{ years}$$

Kepler's three laws of planetary motion can be summarized as follows:

- **Kepler's first law:** Each planet moves around the Sun in an orbit that is an ellipse, with the Sun at one focus of the ellipse.
- **Kepler's second law:** The straight line joining a planet and the Sun sweeps out equal areas in space in equal intervals of time.
- **Kepler's third law:** The square of a planet's orbital period is directly proportional to the cube of the semimajor axis of its orbit.

Kepler's three laws provide a precise geometric description of planetary motion within the framework of the Copernican system. With these tools, it was possible to calculate planetary positions with greatly improved precision. Still, Kepler's laws are purely descriptive: they do not help us understand what forces of nature constrain the planets to follow this particular set of rules. That step was left to Isaac Newton.

EXAMPLE 3.2

Applying Kepler's Third Law

Using the orbital periods and semimajor axes for Venus and Earth that are provided here, calculate P^2 and a^3 , and verify that they obey Kepler's third law. Venus' orbital period is 0.62 year, and its semimajor axis is 0.72 AU. Earth's orbital period is 1.00 year, and its semimajor axis is 1.00 AU.

Solution

We can use the equation for Kepler's third law, $P^2 \propto a^3$. For Venus, $P^2 = 0.62 \times 0.62 = 0.38$ and $a^3 = 0.72 \times 0.72 \times 0.72 = 0.37$ (rounding numbers sometimes causes minor discrepancies like this). The square of the orbital period (0.38) approximates the cube of the semimajor axis (0.37). Therefore, Venus obeys Kepler's third law. For Earth, $P^2 = 1.00 \times 1.00 = 1.00$ and $a^3 = 1.00 \times 1.00 \times 1.00 = 1.00$. The square of the orbital period (1.00) approximates (in this case, equals) the cube of the semimajor axis (1.00). Therefore, Earth obeys Kepler's third law.

Check Your Learning

Using the orbital periods and semimajor axes for Saturn and Jupiter that are provided here, calculate P^2 and a^3 , and verify that they obey Kepler's third law. Saturn's orbital period is 29.46 years, and its semimajor axis is 9.54 AU. Jupiter's orbital period is 11.86 years, and its semimajor axis is 5.20 AU.

Answer:

For Saturn, $P^2 = 29.46 \times 29.46 = 867.9$ and $a^3 = 9.54 \times 9.54 \times 9.54 = 868.3$. The square of the orbital period (867.9) approximates the cube of the semimajor axis (868.3). Therefore, Saturn obeys Kepler's third law.

LINK TO LEARNING



In honor of the scientist who first devised the laws that govern the motions of planets, the team that built the first spacecraft to search for planets orbiting other stars decided to name the probe "Kepler." Visit NASA's Kepler website to learn more about Johannes Kepler's life and his laws of planetary motion. [NASA's Kepler website \(https://openstax.org/l/30nasakepmiss\)](https://openstax.org/l/30nasakepmiss) and follow the links that interest you.

3.2 Newton's Great Synthesis

Learning Objectives

By the end of this section, you will be able to:

- Describe Newton's three laws of motion
- Explain how Newton's three laws of motion relate to momentum
- Define mass, volume, and density and how they differ
- Define angular momentum

It was the genius of Isaac Newton that found a conceptual framework that completely explained the observations and rules assembled by Galileo, Brahe, Kepler, and others. Newton was born in Lincolnshire, England, in the year after Galileo's death (Figure 3.6). Against the advice of his mother, who wanted him to stay home and help with the family farm, he entered Trinity College at Cambridge in 1661 and eight years later was appointed professor of mathematics. Among Newton's contemporaries in England were architect Christopher Wren, authors Aphra Behn and Daniel Defoe, and composer G. F. Handel.