3 ORBITS AND GRAVITY

Figure 3.1 International Space Station. This space habitat and laboratory orbits Earth once every 90 minutes. (credit: modification of work by NASA)

Chapter Outline

Thinking Ahead

How would you find a new planet at the outskirts of our solar system that is too dim to be seen with the unaided eye and is so far away that it moves very slowly among the stars? This was the problem confronting astronomers during the nineteenth century as they tried to pin down a full inventory of our solar system.

If we could look down on the solar system from somewhere out in space, interpreting planetary motions would be much simpler. But the fact is, we must observe the positions of all the other planets from our own moving planet. Scientists of the Renaissance did not know the details of Earth's motions any better than the motions of the other planets. Their problem, as we saw in **Observing the Sky: The Birth of Astronomy**, was that they had to deduce the nature of all planetary motion using only their earthbound observations of the other planets' positions in the sky. To solve this complex problem more fully, better observations and better models of the planetary system were needed.

3.1 THE LAWS OF PLANETARY MOTION

Learning Objectives

By the end of this section, you will be able to:

- Describe how Tycho Brahe and Johannes Kepler contributed to our understanding of how planets move around the Sun
- Explain Kepler's three laws of planetary motion

At about the time that Galileo was beginning his experiments with falling bodies, the efforts of two other scientists dramatically advanced our understanding of the motions of the planets. These two astronomers were the observer Tycho Brahe and the mathematician Johannes Kepler. Together, they placed the speculations of Copernicus on a sound mathematical basis and paved the way for the work of Isaac Newton in the next century.

Tycho Brahe's Observatory

Three years after the publication of Copernicus' *De Revolutionibus*, Tycho Brahe was born to a family of Danish nobility. He developed an early interest in astronomy and, as a young man, made significant astronomical observations. Among these was a careful study of what we now know was an exploding star that flared up to great brilliance in the night sky. His growing reputation gained him the patronage of the Danish King Frederick II, and at the age of 30, Brahe was able to establish a fine astronomical observatory on the North Sea island of Hven (**Figure 3.2**). Brahe was the last and greatest of the pre-telescopic observers in Europe.

(a) (b)

Figure 3.2 Tycho Brahe (1546–1601) and Johannes Kepler (1571–1630). (a) A stylized engraving shows Tycho Brahe using his instruments to measure the altitude of celestial objects above the horizon. The large curved instrument in the foreground allowed him to measure precise angles in the sky. Note that the scene includes hints of the grandeur of Brahe's observatory at Hven. (b) Kepler was a German mathematician and astronomer. His discovery of the basic laws that describe planetary motion placed the heliocentric cosmology of Copernicus on a firm mathematical basis.

At Hven, Brahe made a continuous record of the positions of the Sun, Moon, and planets for almost 20 years. His extensive and precise observations enabled him to note that the positions of the planets varied from those given in published tables, which were based on the work of Ptolemy. These data were extremely valuable, but Brahe didn't have the ability to analyze them and develop a better model than what Ptolemy had published. He was further inhibited because he was an extravagant and cantankerous fellow, and he accumulated enemies among government officials. When his patron, Frederick II, died in 1597, Brahe lost his political base and decided to leave Denmark. He took up residence in Prague, where he became court astronomer to Emperor Rudolf of Bohemia. There, in the year before his death, Brahe found a most able young mathematician, Johannes Kepler, to assist him in analyzing his extensive planetary data.

Johannes Kepler

Johannes Kepler was born into a poor family in the German province of Württemberg and lived much of his life amid the turmoil of the Thirty Years' War (see **Figure 3.2**). He attended university at Tubingen and studied for a theological career. There, he learned the principles of the Copernican system and became converted to the heliocentric hypothesis. Eventually, Kepler went to Prague to serve as an assistant to Brahe, who set him to work trying to find a satisfactory theory of planetary motion—one that was compatible with the long series of observations made at Hven. Brahe was reluctant to provide Kepler with much material at any one time for fear that Kepler would discover the secrets of the universal motion by himself, thereby robbing Brahe of some of the glory. Only after Brahe's death in 1601 did Kepler get full possession of the priceless records. Their study occupied most of Kepler's time for more than 20 years.

Through his analysis of the motions of the planets, Kepler developed a series of principles, now known as *Kepler's three laws,* which described the behavior of planets based on their paths through space. The first two laws of planetary motion were published in 1609 in *The New Astronomy*. Their discovery was a profound step in the development of modern science.

The First Two Laws of Planetary Motion

The path of an object through space is called its **orbit**. Kepler initially assumed that the orbits of planets were circles, but doing so did not allow him to find orbits that were consistent with Brahe's observations. Working with the data for Mars, he eventually discovered that the orbit of that planet had the shape of a somewhat flattened circle, or **ellipse**. Next to the circle, the ellipse is the simplest kind of closed curve, belonging to a family of curves known as *conic sections* (**Figure 3.3**).

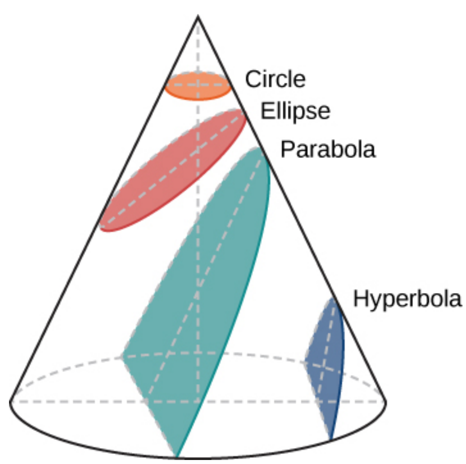


Figure 3.3 Conic Sections. The circle, ellipse, parabola, and hyperbola are all formed by the intersection of a plane with a cone. This is why such curves are called conic sections.

You might recall from math classes that in a circle, the center is a special point. The distance from the center to anywhere on the circle is exactly the same. In an ellipse, the sum of the distance from two special points inside the ellipse to any point on the ellipse is always the same. These two points inside the ellipse are called its foci (singular: **focus**), a word invented for this purpose by Kepler.

This property suggests a simple way to draw an ellipse (**Figure 3.4**). We wrap the ends of a loop of string around two tacks pushed through a sheet of paper into a drawing board, so that the string is slack. If we push a pencil against the string, making the string taut, and then slide the pencil against the string all around the tacks, the curve that results is an ellipse. At any point where the pencil may be, the sum of the distances from the pencil to the two tacks is a constant length—the length of the string. The tacks are at the two foci of the ellipse.

The widest diameter of the ellipse is called its **major axis**. Half this distance—that is, the distance from the center of the ellipse to one end—is the **semimajor axis**, which is usually used to specify the size of the ellipse. For example, the semimajor axis of the orbit of Mars, which is also the planet's average distance from the Sun, is 228 million kilometers.

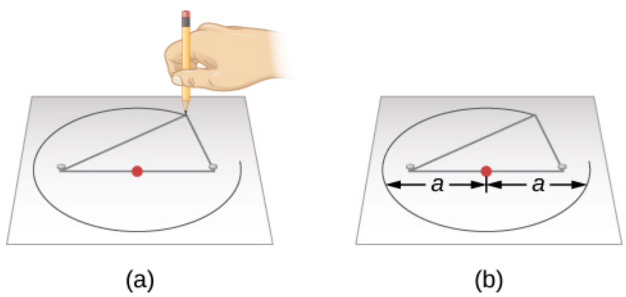


Figure 3.4 Drawing an Ellipse. (a) We can construct an ellipse by pushing two tacks (the white objects) into a piece of paper on a drawing board, and then looping a string around the tacks. Each tack represents a focus of the ellipse, with one of the tacks being the Sun. Stretch the string tight using a pencil, and then move the pencil around the tacks. The length of the string remains the same, so that the sum of the distances from any point on the ellipse to the foci is always constant. (b) In this illustration, each semimajor axis is denoted by a. The distance $2a$ is called the major axis of the ellipse.

The shape (roundness) of an ellipse depends on how close together the two foci are, compared with the major axis. The ratio of the distance between the foci to the length of the major axis is called the **eccentricity** of the ellipse.

If the foci (or tacks) are moved to the same location, then the distance between the foci would be zero. This means that the eccentricity is zero and the ellipse is just a circle; thus, a circle can be called an ellipse of zero eccentricity. In a circle, the semimajor axis would be the radius.

Next, we can make ellipses of various elongations (or extended lengths) by varying the spacing of the tacks (as long as they are not farther apart than the length of the string). The greater the eccentricity, the more elongated is the ellipse, up to a maximum eccentricity of 1.0, when the ellipse becomes "flat," the other extreme from a circle.

The size and shape of an ellipse are completely specified by its semimajor axis and its eccentricity. Using Brahe's data, Kepler found that Mars has an elliptical orbit, with the Sun at one focus (the other focus is empty). The eccentricity of the orbit of Mars is only about 0.1; its orbit, drawn to scale, would be practically indistinguishable from a circle, but the difference turned out to be critical for understanding planetary motions.

Kepler generalized this result in his first law and said that *the orbits of all the planets are ellipses*. Here was a decisive moment in the history of human thought: it was not necessary to have only circles in order to have an acceptable cosmos. The universe could be a bit more complex than the Greek philosophers had wanted it to be.

Kepler's second law deals with the speed with which each planet moves along its ellipse, also known as its **orbital speed**. Working with Brahe's observations of Mars, Kepler discovered that the planet speeds up as it comes closer to the Sun and slows down as it pulls away from the Sun. He expressed the precise form of this relationship by imagining that the Sun and Mars are connected by a straight, elastic line. When Mars is closer to the Sun (positions 1 and 2 in Figure 3.5), the elastic line is not stretched as much, and the planet moves rapidly. Farther from the Sun, as in positions 3 and 4, the line is stretched a lot, and the planet does not move so fast. As Mars travels in its elliptical orbit around the Sun, the elastic line sweeps out areas of the ellipse as it moves (the colored regions in our figure). Kepler found that in equal intervals of time (t), the areas swept out in space by this imaginary line are always equal; that is, the area of the region B from 1 to 2 is the same as that of region A from 3 to 4.

If a planet moves in a circular orbit, the elastic line is always stretched the same amount and the planet moves at a constant speed around its orbit. But, as Kepler discovered, in most orbits that speed of a planet orbiting its star (or moon orbiting its planet) tends to vary because the orbit is elliptical.

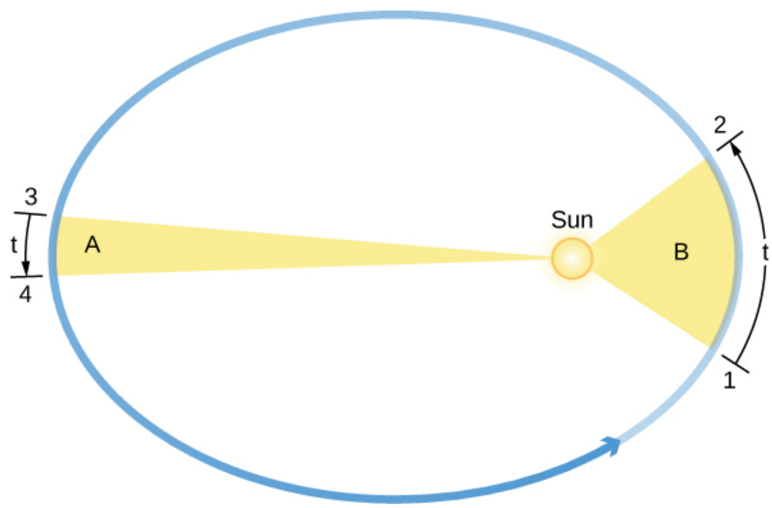


Figure 3.5 Kepler's Second Law: The Law of Equal Areas. The orbital speed of a planet traveling around the Sun (the circular object inside the ellipse) varies in such a way that in equal intervals of time (t), a line between the Sun and a planet sweeps out equal areas (A and B). Note that the eccentricities of the planets' orbits in our solar system are substantially less than shown here.

Kepler's Third Law

Kepler's first two laws of planetary motion describe the shape of a planet's orbit and allow us to calculate the speed of its motion at any point in the orbit. Kepler was pleased to have discovered such fundamental rules, but they did not satisfy his quest to fully understand planetary motions. He wanted to know why the orbits of the planets were spaced as they are and to find a mathematical pattern in their movements—a "harmony of the spheres" as he called it. For many years he worked to discover mathematical relationships governing planetary spacing and the time each planet took to go around the Sun.

In 1619, Kepler discovered a basic relationship to relate the planets' orbits to their relative distances from the Sun. We define a planet's **orbital period**, (***P***), as the time it takes a planet to travel once around the Sun. Also, recall that a planet's semimajor axis, *a,* is equal to its average distance from the Sun. The relationship, now known as *Kepler's third law*, says that a planet's orbital period squared is proportional to the semimajor axis of its orbit cubed, or

$$P^2 \propto a^3$$

When *P* (the orbital period) is measured in years, and *a* is expressed in a quantity known as an **astronomical unit (AU)**, the two sides of the formula are not only proportional but equal. One AU is the average distance between Earth and the Sun and is approximately equal to 1.5 × 10^8 kilometers. In these units,

$$P^2 = a^3$$

Kepler's third law applies to all objects orbiting the Sun, including Earth, and provides a means for calculating their relative distances from the Sun from the time they take to orbit. Let's look at a specific example to illustrate how useful Kepler's third law is.

For instance, suppose you time how long Mars takes to go around the Sun (in Earth years). Kepler's third law can then be used to calculate Mars' average distance from the Sun. Mars' orbital period (1.88 Earth years) squared, or P^2, is 1.88^2 = 3.53, and according to the equation for Kepler's third law, this equals the cube of its semimajor axis, or a^3. So what number must be cubed to give 3.53? The answer is 1.52 (since 1.52 × 1.52 × 1.52 = 3.53). Thus, Mars' semimajor axis in astronomical units must be 1.52 AU. In other words, to go around the Sun in a little less than two years, Mars must be about 50% (half again) as far from the Sun as Earth is.

EXAMPLE 3.1

Calculating Periods

Imagine an object is traveling around the Sun. What would be the orbital period of the object if its orbit has a semimajor axis of 50 AU?

Solution

From Kepler's third law, we know that (when we use units of years and AU)

$$P^2 = a^3$$

If the object's orbit has a semimajor axis of 50 AU (a = 50), we can cube 50 and then take the square root of the result to get P:

$$\begin{aligned} P &= \sqrt{a^3} \\ P &= \sqrt{50 \times 50 \times 50} = \sqrt{125{,}000} = 353.6 \text{ years} \end{aligned}$$

Therefore, the orbital period of the object is about 350 years. This would place our hypothetical object beyond the orbit of Pluto.

Check Your Learning

What would be the orbital period of an asteroid (a rocky chunk between Mars and Jupiter) with a semimajor axis of 3 AU?

Answer:

$$P = \sqrt{3 \times 3 \times 3} = \sqrt{27} = 5.2 \text{ years}$$

Kepler's three laws of planetary motion can be summarized as follows:

- **Kepler's first law**: Each planet moves around the Sun in an orbit that is an ellipse, with the Sun at one focus of the ellipse.
- **Kepler's second law**: The straight line joining a planet and the Sun sweeps out equal areas in space in equal intervals of time.

- **Kepler's third law**: The square of a planet's orbital period is directly proportional to the cube of the semimajor axis of its orbit.

Kepler's three laws provide a precise geometric description of planetary motion within the framework of the Copernican system. With these tools, it was possible to calculate planetary positions with greatly improved precision. Still, Kepler's laws are purely descriptive: they do not help us understand what forces of nature constrain the planets to follow this particular set of rules. That step was left to Isaac Newton.

EXAMPLE 3.2

Applying Kepler's Third Law

Using the orbital periods and semimajor axes for Venus and Earth that are provided here, calculate P^2 and a^3, and verify that they obey Kepler's third law. Venus' orbital period is 0.62 year, and its semimajor axis is 0.72 AU. Earth's orbital period is 1.00 year, and its semimajor axis is 1.00 AU.

Solution

We can use the equation for Kepler's third law, $P^2 \propto a^3$. For Venus, $P^2 = 0.62 \times 0.62 = 0.38$ and $a^3 = 0.72 \times 0.72 \times 0.72 = 0.37$ (rounding numbers sometimes causes minor discrepancies like this). The square of the orbital period (0.38) approximates the cube of the semimajor axis (0.37). Therefore, Venus obeys Kepler's third law. For Earth, $P^2 = 1.00 \times 1.00 = 1.00$ and $a^3 = 1.00 \times 1.00 \times 1.00 = 1.00$. The square of the orbital period (1.00) approximates (in this case, equals) the cube of the semimajor axis (1.00). Therefore, Earth obeys Kepler's third law.

Check Your Learning

Using the orbital periods and semimajor axes for Saturn and Jupiter that are provided here, calculate P^2 and a^3, and verify that they obey Kepler's third law. Saturn's orbital period is 29.46 years, and its semimajor axis is 9.54 AU. Jupiter's orbital period is 11.86 years, and its semimajor axis is 5.20 AU.

Answer:

For Saturn, $P^2 = 29.46 \times 29.46 = 867.9$ and $a^3 = 9.54 \times 9.54 \times 9.54 = 868.3$. The square of the orbital period (867.9) approximates the cube of the semimajor axis (868.3). Therefore, Saturn obeys Kepler's third law.

LINK TO LEARNING

In honor of the scientist who first devised the laws that govern the motions of planets, the team that built the first spacecraft to search for planets orbiting other stars decided to name the probe "Kepler." To learn more about Johannes Kepler's life and his laws of planetary motion, as well as lots of information on the Kepler Mission, visit **NASA's Kepler website (https://openstaxcollege.org/l/30nasakepmiss)** and follow the links that interest you.

3.2 NEWTON'S GREAT SYNTHESIS

Learning Objectives

By the end of this section, you will be able to:

- Describe Newton's three laws of motion
- Explain how Newton's three laws of motion relate to momentum
- Define mass, volume, and density and how they differ
- Define angular momentum

It was the genius of Isaac Newton that found a conceptual framework that completely explained the observations and rules assembled by Galileo, Brahe, Kepler, and others. Newton was born in Lincolnshire, England, in the year after Galileo's death (Figure 3.6). Against the advice of his mother, who wanted him to stay home and help with the family farm, he entered Trinity College at Cambridge in 1661 and eight years later was appointed professor of mathematics. Among Newton's contemporaries in England were architect Christopher Wren, authors Aphra Behn and Daniel Defoe, and composer G. F. Handel.

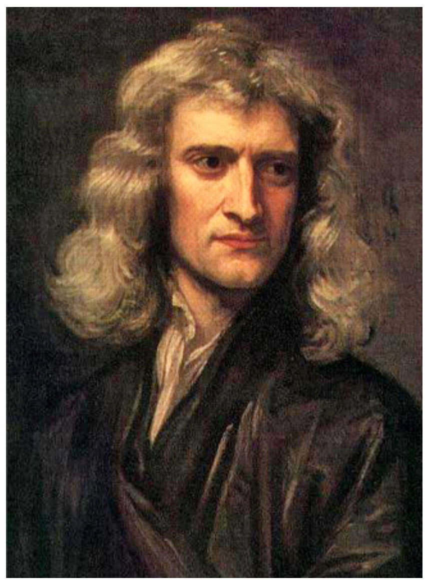


Figure 3.6 Isaac Newton (1643–1727), 1689 Portrait by Sir Godfrey Kneller. Isaac Newton's work on the laws of motion, gravity, optics, and mathematics laid the foundations for much of physical science.

Newton's Laws of Motion

As a young man in college, Newton became interested in natural philosophy, as science was then called. He worked out some of his first ideas on machines and optics during the plague years of 1665 and 1666, when students were sent home from college. Newton, a moody and often difficult man, continued to work on his ideas in private, even inventing new mathematical tools to help him deal with the complexities involved. Eventually, his friend Edmund Halley (profiled in **Comets and Asteroids: Debris of the Solar System**) prevailed on him to collect and publish the results of his remarkable investigations on motion and gravity. The result was a volume that set out the underlying system of the physical world, *Philosophiae Naturalis Principia Mathematica*. The *Principia*, as the book is generally known, was published at Halley's expense in 1687.

At the very beginning of the *Principia*, Newton proposes three laws that would govern the motions of all objects:

- **Newton's first law**: Every object will continue to be in a state of rest or move at a constant speed in a straight line unless it is compelled to change by an outside force.
- **Newton's second law**: The change of motion of a body is proportional to and in the direction of the force acting on it.
- **Newton's third law**: For every action there is an equal and opposite reaction (*or:* the mutual actions of two bodies upon each other are always equal and act in opposite directions).

In the original Latin, the three laws contain only 59 words, but those few words set the stage for modern science. Let us examine them more carefully.

Interpretation of Newton's Laws

Newton's first law is a restatement of one of Galileo's discoveries, called the *conservation of momentum*. The law states that in the absence of any outside influence, there is a measure of a body's motion, called its **momentum**, that remains unchanged. You may have heard the term momentum used in everyday expressions, such as "This bill in Congress has a lot of momentum; it's going to be hard to stop."

Newton's first law is sometimes called the *law of inertia*, where inertia is the tendency of objects (and legislatures) to keep doing what they are already doing. In other words, a stationary object stays put, and a moving object keeps moving unless some force intervenes.

Let's define the precise meaning of momentum—it depends on three factors: (1) speed—how fast a body moves (zero if it is stationary), (2) the direction of its motion, and (3) its mass—a measure of the amount of matter in a body, which we will discuss later. Scientists use the term **velocity** to describe the speed and direction of motion. For example, 20 kilometers per hour due south is velocity, whereas 20 kilometers per hour just by itself is speed. Momentum then can be defined as an object's mass times its velocity.

It's not so easy to see this rule in action in the everyday world because of the many forces acting on a body at any one time. One important force is friction, which generally slows things down. If you roll a ball along the sidewalk, it eventually comes to a stop because the sidewalk exerts a rubbing force on the ball. But in the space between the stars, where there is so little matter that friction is insignificant, objects can in fact continue to move (to coast) indefinitely.

The momentum of a body can change only under the action of an outside influence. Newton's second law expresses *force* in terms of its ability to change momentum with time. A force (a push or a pull) has both size and direction. When a force is applied to a body, the momentum changes in the direction of the applied force. This means that a force is required to change either the speed or the direction of a body, or both—that is, to start it moving, to speed it up, to slow it down, to stop it, or to change its direction.

As you learned in **Observing the Sky: The Birth of Astronomy**, the rate of change in an object's velocity is called *acceleration*. Newton showed that the acceleration of a body was proportional to the force being applied to it. Suppose that after a long period of reading, you push an astronomy book away from you on a long, smooth table. (We use a smooth table so we can ignore friction.) If you push the book steadily, it will continue to speed up as long as you are pushing it. The harder you push the book, the larger its acceleration will be. How much a force will accelerate an object is also determined by the object's mass. If you kept pushing a pen with the same force with which you pushed the textbook, the pen—having less mass—would be accelerated to a greater speed.

Newton's third law is perhaps the most profound of the rules he discovered. Basically, it is a generalization of the first law, but it also gives us a way to define mass. If we consider a system of two or more objects isolated from outside influences, Newton's first law says that the total momentum of the objects should remain constant. Therefore, any change of momentum within the system must be balanced by another change that is equal and opposite so that the momentum of the entire system is not changed.

This means that forces in nature do not occur alone: we find that in each situation there is always a *pair* of forces that are equal to and opposite each other. If a force is exerted on an object, it must be exerted by something else, and the object will exert an equal and opposite force back on that something. We can look at a simple example to demonstrate this.

Suppose that a daredevil astronomy student—and avid skateboarder—wants to jump from his second-story dorm window onto his board below (we don’t recommend trying this!). The force pulling him down after jumping (as we will see in the next section) is the force of gravity between him and Earth. Both he and Earth must experience the same total change of momentum because of the influence of these mutual forces. So, both the student and Earth are accelerated by each other’s pull. However, the student does much more of the moving. Because Earth has enormously greater mass, it can experience the same change of momentum by accelerating only a very small amount. Things fall toward Earth all the time, but the acceleration of our planet as a result is far too small to be measured.

A more obvious example of the mutual nature of forces between objects is familiar to all who have batted a baseball. The recoil you feel as you swing your bat shows that the ball exerts a force on it during the impact, just as the bat does on the ball. Similarly, when a rifle you are bracing on your shoulder is discharged, the force pushing the bullet out of the muzzle is equal to the force pushing backward upon the gun and your shoulder.

This is the principle behind jet engines and rockets: the force that discharges the exhaust gases from the rear of the rocket is accompanied by the force that pushes the rocket forward. The exhaust gases need not push against air or Earth; a rocket actually operates best in a vacuum (**Figure 3.7**).

Figure 3.7 Demonstrating Newton’s Third Law. The U.S. Space Shuttle (here launching *Discovery*), powered by three fuel engines burning liquid oxygen and liquid hydrogen, with two solid fuel boosters, demonstrates Newton’s third law. (credit: modification of work by NASA)

LINK TO LEARNING

For more about Isaac Newton’s life and work, check out this **timeline page (https://openstaxcollege.org/l/30IsaacNewTime)** with snapshots from his career, produced by the British Broadcasting Corporation (BBC).

Mass, Volume, and Density

Before we go on to discuss Newton's other work, we want to take a brief look at some terms that will be important to sort out clearly. We begin with *mass,* which is a measure of the amount of material within an object.

The *volume* of an object is the measure of the physical space it occupies. Volume is measured in cubic units, such as cubic centimeters or liters. The volume is the "size" of an object. A penny and an inflated balloon may both have the same mass, but they have very different volumes. The reason is that they also have very different *densities*, which is a measure of how much mass there is per unit volume. Specifically, **density** is the mass divided by the volume. Note that in everyday language we often use "heavy" and "light" as indications of density (rather than weight) as, for instance, when we say that iron is heavy or that whipped cream is light.

The units of density that will be used in this book are grams per cubic centimeter (g/cm^3).[1] If a block of some material has a mass of 300 grams and a volume of 100 cm^3, its density is 3 g/cm^3. Familiar materials span a considerable range in density, from artificial materials such as plastic insulating foam (less than 0.1 g/cm^3) to gold (19.3 g/cm^3). Table 3.1 gives the densities of some familiar materials. In the astronomical universe, much more remarkable densities can be found, all the way from a comet's tail (10^{-16} g/cm^3) to a collapsed "star corpse" called a neutron star (10^{15} g/cm^3).

Densities of Common Materials

Material	Density (g/cm^3)
Gold	19.3
Lead	11.3
Iron	7.9
Earth (bulk)	5.5
Rock (typical)	2.5
Water	1
Wood (typical)	0.8
Insulating foam	0.1
Silica gel	0.02

Table 3.1

To sum up, mass is *how much*, volume is *how big*, and density is *how tightly packed*.

1 Generally we use standard metric (or SI) units in this book. The proper metric unit of density in that system is kg/m^3. But to most people, g/cm^3 provides a more meaningful unit because the density of water is exactly 1 g/cm^3, and this is useful information for comparison. Density expressed in g/cm^3 is sometimes called specific density or specific weight.

LINK TO LEARNING

You can play with a **simple animation (https://openstaxcollege.org/l/30phetsimdenmas)** demonstrating the relationship between the concepts of density, mass, and volume, and find out why objects like wood float in water.

Angular Momentum

A concept that is a bit more complex, but important for understanding many astronomical objects, is **angular momentum**, which is a measure of the rotation of a body as it revolves around some fixed point (an example is a planet orbiting the Sun). The angular momentum of an object is defined as the product of its mass, its velocity, and its distance from the fixed point around which it revolves.

If these three quantities remain constant—that is, if the motion of a particular object takes place at a constant velocity at a fixed distance from the spin center—then the angular momentum is also a constant. Kepler's second law is a consequence of the *conservation of angular momentum*. As a planet approaches the Sun on its elliptical orbit and the distance to the spin center decreases, the planet speeds up to conserve the angular momentum. Similarly, when the planet is farther from the Sun, it moves more slowly.

The conservation of angular momentum is illustrated by figure skaters, who bring their arms and legs in to spin more rapidly, and extend their arms and legs to slow down (**Figure 3.8**). You can duplicate this yourself on a well-oiled swivel stool by starting yourself spinning slowly with your arms extended and then pulling your arms in. Another example of the conservation of angular momentum is a shrinking cloud of dust or a star collapsing on itself (both are situations that you will learn about as you read on). As material moves to a lesser distance from the spin center, the speed of the material increases to conserve angular momentum.

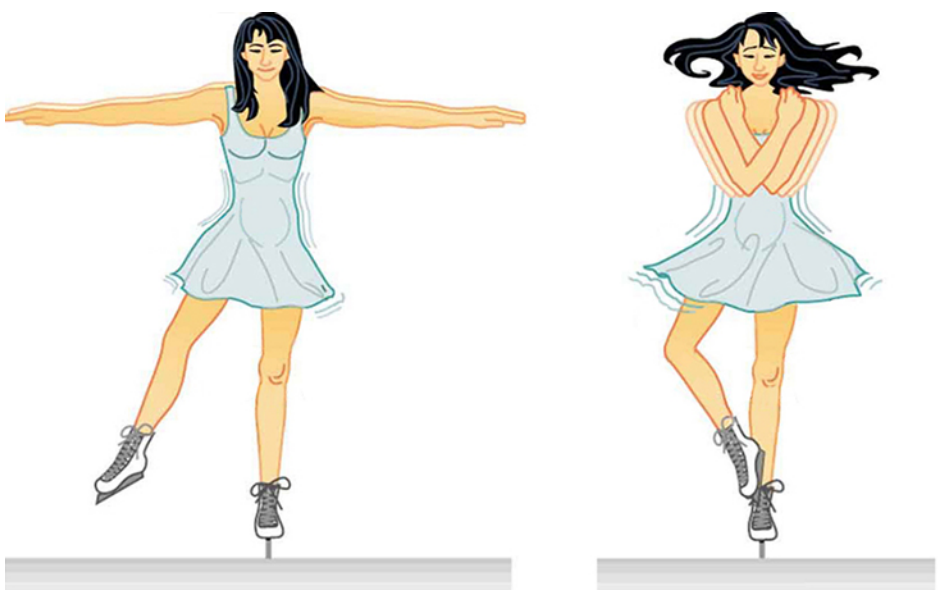


Figure 3.8 Conservation of Angular Momentum. When a spinning figure skater brings in her arms, their distance from her spin center is smaller, so her speed increases. When her arms are out, their distance from the spin center is greater, so she slows down.

3.3 NEWTON'S UNIVERSAL LAW OF GRAVITATION

Learning Objectives

By the end of this section, you will be able to:

- Explain what determines the strength of gravity
- Describe how Newton's universal law of gravitation extends our understanding of Kepler's laws

Newton's laws of motion show that objects at rest will stay at rest and those in motion will continue moving uniformly in a straight line unless acted upon by a force. Thus, it is the *straight line* that defines the most natural state of motion. But the planets move in ellipses, not straight lines; therefore, some force must be bending their paths. That force, Newton proposed, was **gravity**.

In Newton's time, gravity was something associated with Earth alone. Everyday experience shows us that Earth exerts a gravitational force upon objects at its surface. If you drop something, it accelerates toward Earth as it falls. Newton's insight was that Earth's gravity might extend as far as the Moon and produce the force required to curve the Moon's path from a straight line and keep it in its orbit. He further hypothesized that gravity is not limited to Earth, but that there is a general force of attraction between all material bodies. If so, the attractive force between the Sun and each of the planets could keep them in their orbits. (This may seem part of our everyday thinking today, but it was a remarkable insight in Newton's time.)

Once Newton boldly hypothesized that there was a universal attraction among all bodies everywhere in space, he had to determine the exact nature of the attraction. The precise mathematical description of that gravitational force had to dictate that the planets move exactly as Kepler had described them to (as expressed in Kepler's three laws). Also, that gravitational force had to predict the correct behavior of falling bodies on Earth, as observed by Galileo. How must the force of gravity depend on distance in order for these conditions to be met?

The answer to this question required mathematical tools that had not yet been developed, but this did not deter Isaac Newton, who invented what we today call calculus to deal with this problem. Eventually he was able to conclude that the magnitude of the force of gravity must decrease with increasing distance between the Sun and a planet (or between any two objects) in proportion to the inverse square of their separation. In other words, if a planet were twice as far from the Sun, the force would be $(1/2)^2$, or 1/4 as large. Put the planet three times farther away, and the force is $(1/3)^2$, or 1/9 as large.

Newton also concluded that the gravitational attraction between two bodies must be proportional to their masses. The more mass an object has, the stronger the pull of its gravitational force. The gravitational attraction between any two objects is therefore given by one of the most famous equations in all of science:

$$F_{\text{gravity}} = G\frac{M_1 M_2}{R^2}$$

where F_{gravity} is the gravitational force between two objects, M_1 and M_2 are the masses of the two objects, and R is their separation. G is a constant number known as the *universal gravitational constant*, and the equation itself symbolically summarizes Newton's *universal law of gravitation*. With such a force and the laws of motion, Newton was able to show mathematically that the only orbits permitted were exactly those described by Kepler's laws.

Newton's universal law of gravitation works for the planets, but is it really universal? The gravitational theory should also predict the observed acceleration of the Moon toward Earth as it orbits Earth, as well as of any object (say, an apple) dropped near Earth’s surface. The falling of an apple is something we can measure quite easily, but can we use it to predict the motions of the Moon?

Recall that according to Newton’s second law, forces cause acceleration. Newton’s universal law of gravitation says that the force acting upon (and therefore the acceleration of) an object toward Earth should be inversely proportional to the square of its distance from the center of Earth. Objects like apples at the surface of Earth, at a distance of one Earth-radius from the center of Earth, are observed to accelerate downward at 9.8 meters per second per second (9.8 m/s^2).

It is this force of gravity on the surface of Earth that gives us our sense of *weight.* Unlike your mass, which would remain the same on any planet or moon, your weight depends on the local force of gravity. So you would weigh less on Mars and the Moon than on Earth, even though there is no change in your mass. (Which means you would still have to go easy on the desserts in the college cafeteria when you got back!)

The Moon is 60 Earth radii away from the center of Earth. If gravity (and the acceleration it causes) gets weaker with distance squared, the acceleration the Moon experiences should be a lot less than for the apple. The acceleration should be $(1/60)^2$ = 1/3600 (or 3600 times less—about 0.00272 m/s^2. This is precisely the observed acceleration of the Moon in its orbit. (As we shall see, the Moon does not fall *to* Earth with this acceleration, but falls *around* Earth.) Imagine the thrill Newton must have felt to realize he had discovered, and verified, a law that holds for Earth, apples, the Moon, and, as far as he knew, everything in the universe.

EXAMPLE 3.3

Calculating Weight

By what factor would a person’s weight at the surface of Earth change if Earth had its present mass but eight times its present volume?

Solution

With eight times the volume, Earth’s radius would double. This means the gravitational force at the surface would reduce by a factor of $(1/2)^2 = 1/4$, so a person would weigh only one-fourth as much.

Check Your Learning

By what factor would a person’s weight at the surface of Earth change if Earth had its present size but only one-third its present mass?

Answer:

With one-third its present mass, the gravitational force at the surface would reduce by a factor of 1/3, so a person would weight only one-third as much.

Gravity is a “built-in” property of mass. Whenever there are masses in the universe, they will interact via the force of gravitational attraction. The more mass there is, the greater the force of attraction. Here on Earth, the largest concentration of mass is, of course, the planet we stand on, and its pull dominates the gravitational interactions we experience. But everything with mass attracts everything else with mass anywhere in the universe.

Newton’s law also implies that gravity never becomes zero. It quickly gets weaker with distance, but it continues to act to some degree no matter how far away you get. The pull of the Sun is stronger at Mercury than at Pluto, but it can be felt far beyond Pluto, where astronomers have good evidence that it continuously makes enormous numbers of smaller icy bodies move around huge orbits. And the Sun's gravitational pull joins with the pull of billions of others stars to create the gravitational pull of our Milky Way Galaxy. That force, in turn, can make other smaller galaxies orbit around the Milky Way, and so on.

Why is it then, you may ask, that the astronauts aboard the Space Shuttle appear to have no gravitational forces acting on them when we see images on television of the astronauts and objects floating in the spacecraft? After all, the astronauts in the shuttle are only a few hundred kilometers above the surface of Earth, which is not a significant distance compared to the size of Earth, so gravity is certainly not a great deal weaker that much farther away. The astronauts feel "weightless" (meaning that they don't feel the gravitational force acting on them) for the same reason that passengers in an elevator whose cable has broken or in an airplane whose engines no longer work feel weightless: they are falling (**Figure 3.9**).[2]

Figure 3.9 Astronauts in Free Fall. While in space, astronauts are falling freely, so they experience "weightlessness." Clockwise from top left: Tracy Caldwell Dyson (NASA), Naoko Yamzaki (JAXA), Dorothy Metcalf-Lindenburger (NASA), and Stephanie Wilson (NASA). (credit: NASA)

When *falling*, they are in free fall and accelerate at the same rate as everything around them, including their spacecraft or a camera with which they are taking photographs of Earth. When doing so, astronauts experience no additional forces and therefore feel "weightless." Unlike the falling elevator passengers, however, the astronauts are falling *around* Earth, not *to* Earth; as a result they will continue to fall and are said to be "in orbit" around Earth (see the next section for more about orbits).

Orbital Motion and Mass

Kepler's laws describe the orbits of the objects whose motions are described by Newton's laws of motion and

2 In the film *Apollo 13*, the scenes in which the astronauts were "weightless" were actually filmed in a falling airplane. As you might imagine, the plane fell for only short periods before the engines engaged again.

the law of gravity. Knowing that gravity is the force that attracts planets toward the Sun, however, allowed Newton to rethink Kepler's third law. Recall that Kepler had found a relationship between the orbital period of a planet's revolution and its distance from the Sun. But Newton's formulation introduces the additional factor of the masses of the Sun (M_1) and the planet (M_2), both expressed in units of the Sun's mass. Newton's universal law of gravitation can be used to show mathematically that this relationship is actually

$$a^3 = \left(M_1 + M_2\right) \times P^2$$

where a is the semimajor axis and P is the orbital period.

How did Kepler miss this factor? In units of the Sun's mass, the mass of the Sun is 1, and in units of the Sun's mass, the mass of a typical planet is a negligibly small factor. This means that the sum of the Sun's mass and a planet's mass, ($M_1 + M_2$), is very, very close to 1. This makes Newton's formula appear almost the same as Kepler's; the tiny mass of the planets compared to the Sun is the reason that Kepler did not realize that both masses had to be included in the calculation. There are many situations in astronomy, however, in which we *do* need to include the two mass terms—for example, when two stars or two galaxies orbit each other.

Including the mass term allows us to use this formula in a new way. If we can measure the motions (distances and orbital periods) of objects acting under their mutual gravity, then the formula will permit us to deduce their masses. For example, we can calculate the mass of the Sun by using the distances and orbital periods of the planets, or the mass of Jupiter by noting the motions of its moons.

Indeed, Newton's reformulation of Kepler's third law is one of the most powerful concepts in astronomy. Our ability to deduce the masses of objects from their motions is key to understanding the nature and evolution of many astronomical bodies. We will use this law repeatedly throughout this text in calculations that range from the orbits of comets to the interactions of galaxies.

EXAMPLE 3.4

Calculating the Effects of Gravity

A planet like Earth is found orbiting its star at a distance of 1 AU in 0.71 Earth-year. Can you use Newton's version of Kepler's third law to find the mass of the star? (Remember that compared to the mass of a star, the mass of an earthlike planet can be considered negligible.)

Solution

In the formula $a^3 = (M_1 + M_2) \times P^2$, the factor $M_1 + M_2$ would now be approximately equal to M_1 (the mass of the star), since the planet's mass is so small by comparison. Then the formula becomes $a^3 = M_1 \times P^2$, and we can solve for M_1:

$$M_1 = \frac{a^3}{P^2}$$

Since $a = 1$, $a^3 = 1$, so

$$M_1 = \frac{1}{P^2} = \frac{1}{0.71^2} = \frac{1}{0.5} = 2$$

So the mass of the star is twice the mass of our Sun. (Remember that this way of expressing the law has units in terms of Earth and the Sun, so masses are expressed in units of the mass of our Sun.)

Check Your Learning

Suppose a star with twice the mass of our Sun had an earthlike planet that took 4 years to orbit the star. At what distance (semimajor axis) would this planet orbit its star?

Answer:

Again, we can neglect the mass of the planet. So M_1 = 2 and P = 4 years. The formula is $a^3 = M_1 \times P^2$, so $a^3 = 2 \times 4^2 = 2 \times 16 = 32$. So a is the cube root of 32. To find this, you can just ask Google, "What is the cube root of 32?" and get the answer 3.2 AU.

LINK TO LEARNING

You might like to try a **simulation (https://openstaxcollege.org/l/30phetsimsunear)** that lets you move the Sun, Earth, Moon, and space station to see the effects of changing their distances on their gravitational forces and orbital paths. You can even turn off gravity and see what happens.

3.4 ORBITS IN THE SOLAR SYSTEM

Learning Objectives

By the end of this section, you will be able to:

- Compare the orbital characteristics of the planets in the solar system
- Compare the orbital characteristics of asteroids and comets in the solar system

Recall that the path of an object under the influence of gravity through space is called its orbit, whether that object is a spacecraft, planet, star, or galaxy. An orbit, once determined, allows the future positions of the object to be calculated.

Two points in any orbit in our solar system have been given special names. The place where the planet is closest to the Sun (*helios* in Greek) and moves the fastest is called the **perihelion** of its orbit, and the place where it is farthest away and moves the most slowly is the **aphelion**. For the Moon or a satellite orbiting Earth (*gee* in Greek), the corresponding terms are **perigee** and **apogee**. (In this book, we use the word *moon* for a natural object that goes around a planet and the word **satellite** to mean a human-made object that revolves around a planet.)

Orbits of the Planets

Today, Newton's work enables us to calculate and predict the orbits of the planets with marvelous precision. We know eight planets, beginning with Mercury closest to the Sun and extending outward to Neptune. The average orbital data for the planets are summarized in Table 3.2. (Ceres is the largest of the *asteroids,* now considered a dwarf planet.)

According to Kepler's laws, Mercury must have the shortest orbital period (88 Earth-days); thus, it has the highest orbital speed, averaging 48 kilometers per second. At the opposite extreme, Neptune has a period of

165 years and an average orbital speed of just 5 kilometers per second.

All the planets have orbits of rather low eccentricity. The most eccentric orbit is that of Mercury (0.21); the rest have eccentricities smaller than 0.1. It is fortunate that among the rest, Mars has an eccentricity greater than that of many of the other planets. Otherwise the pre-telescopic observations of Brahe would not have been sufficient for Kepler to deduce that its orbit had the shape of an ellipse rather than a circle.

The planetary orbits are also confined close to a common plane, which is near the plane of Earth's orbit (called the ecliptic). The strange orbit of the dwarf planet Pluto is inclined about 17° to the ecliptic, and that of the dwarf planet Eris (orbiting even farther away from the Sun than Pluto) by 44°, but all the major planets lie within 10° of the common plane of the solar system.

LINK TO LEARNING

You can use an **orbital simulator (https://openstaxcollege.org/l/30phetorbsim)** to design your own mini solar system with up to four bodies. Adjust masses, velocities, and positions of the planets, and see what happens to their orbits as a result.

Orbits of Asteroids and Comets

In addition to the eight planets, there are many smaller objects in the solar system. Some of these are moons (natural satellites) that orbit all the planets except Mercury and Venus. In addition, there are two classes of smaller objects in heliocentric orbits: *asteroids* and *comets*. Both asteroids and comets are believed to be small chunks of material left over from the formation process of the solar system.

In general, asteroids have orbits with smaller semimajor axes than do comets (**Figure 3.10**). The majority of them lie between 2.2 and 3.3 AU, in the region known as the **asteroid belt** (see **Comets and Asteroids: Debris of the Solar System**). As you can see in **Table 3.2**, the asteroid belt (represented by its largest member, Ceres) is in the middle of a gap between the orbits of Mars and Jupiter. It is because these two planets are so far apart that stable orbits of small bodies can exist in the region between them.

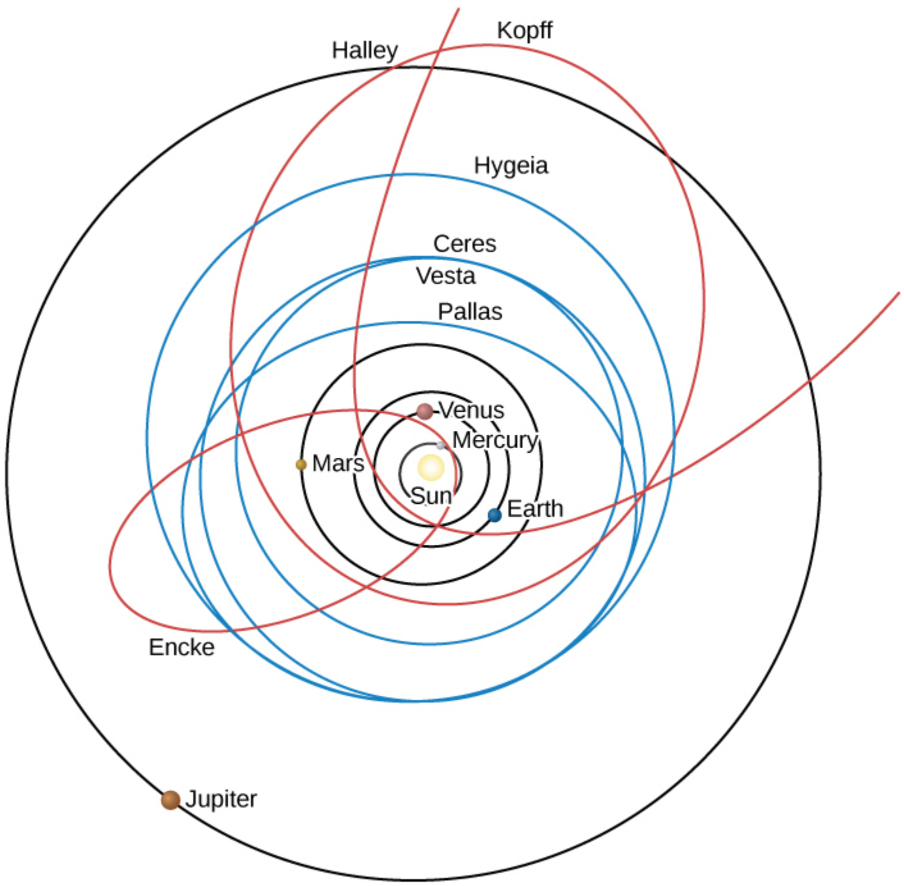


Figure 3.10 Solar System Orbits. We see the orbits of typical comets and asteroids compared with those of the planets Mercury, Venus, Earth, Mars, and Jupiter (black circles). Shown in red are three comets: Halley, Kopff, and Encke. In blue are the four largest asteroids: Ceres, Pallas, Vesta, and Hygeia.

Orbital Data for the Planets

Planet	Semimajor Axis (AU)	Period (y)	Eccentricity
Mercury	0.39	0.24	0.21
Venus	0.72	0.6	0.01
Earth	1	1.00	0.02
Mars	1.52	1.88	0.09
(Ceres)	2.77	4.6	0.08
Jupiter	5.20	11.86	0.05
Saturn	9.54	29.46	0.06
Uranus	19.19	84.01	0.05

Table 3.2

Orbital Data for the Planets

Planet	Semimajor Axis (AU)	Period (y)	Eccentricity
Neptune	30.06	164.82	0.01

Table 3.2

Comets generally have orbits of larger size and greater eccentricity than those of the asteroids. Typically, the eccentricity of their orbits is 0.8 or higher. According to Kepler's second law, therefore, they spend most of their time far from the Sun, moving very slowly. As they approach perihelion, the comets speed up and whip through the inner parts of their orbits more rapidly.

3.5 MOTIONS OF SATELLITES AND SPACECRAFT

Learning Objectives

By the end of this section, you will be able to:

- Explain how an object (such as a satellite) can be put into orbit around Earth
- Explain how an object (such as a planetary probe) can escape from orbit

Newton's universal law of gravitation and Kepler's laws describe the motions of Earth satellites and interplanetary spacecraft as well as the planets. Sputnik, the first artificial Earth satellite, was launched by what was then called the Soviet Union on October 4, 1957. Since that time, thousands of satellites have been placed into orbit around Earth, and spacecraft have also orbited the Moon, Venus, Mars, Jupiter, Saturn, and a number of asteroids and comets.

Once an artificial satellite is in orbit, its behavior is no different from that of a natural satellite, such as our Moon. If the satellite is high enough to be free of atmospheric friction, it will remain in orbit forever. However, although there is no difficulty in maintaining a satellite once it is in orbit, a great deal of energy is required to lift the spacecraft off Earth and accelerate it to orbital speed.

To illustrate how a satellite is launched, imagine a gun firing a bullet horizontally from the top of a high mountain, as in **Figure 3.11**, which has been adapted from a similar diagram by Newton. Imagine, further, that the friction of the air could be removed and that nothing gets in the bullet's way. Then the only force that acts on the bullet after it leaves the muzzle is the gravitational force between the bullet and Earth.

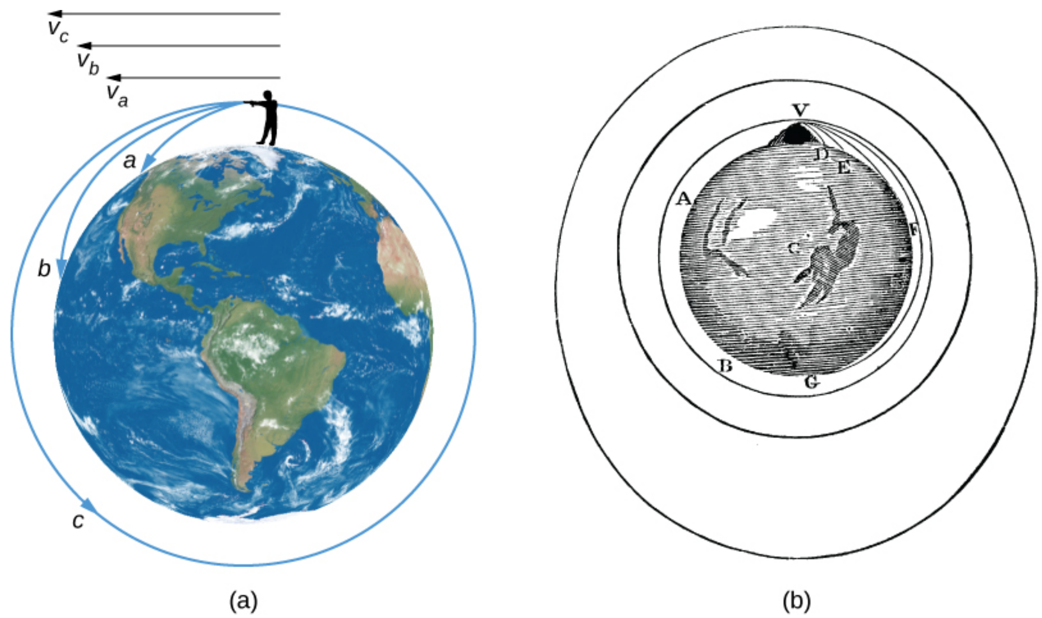


Figure 3.11 Firing a Bullet into Orbit. (a) For paths *a* and *b*, the velocity is not enough to prevent gravity from pulling the bullet back to Earth; in case *c*, the velocity allows the bullet to fall completely around Earth. (b) This diagram by Newton in his *De Mundi Systemate*, 1731 edition, illustrates the same concept shown in (a).

If the bullet is fired with a velocity we can call v_a, the gravitational force acting upon it pulls it downward toward Earth, where it strikes the ground at point *a*. However, if it is given a higher muzzle velocity, v_b, its higher speed carries it farther before it hits the ground at point *b*.

If our bullet is given a high enough muzzle velocity, v_c, the curved surface of Earth causes the ground to remain the same distance from the bullet so that the bullet falls *around* Earth in a complete circle. The speed needed to do this—called the circular satellite velocity—is about 8 kilometers per second, or about 17,500 miles per hour in more familiar units.

Each year, more than 50 new satellites are launched into orbit by such nations as Russia, the United States, China, Japan, India, and Israel, as well as by the European Space Agency (ESA), a consortium of European nations (**Figure 3.12**). Today, these satellites are used for weather tracking, ecology, global positioning systems, communications, and military purposes, to name a few uses. Most satellites are launched into low Earth orbit, since this requires the minimum launch energy. At the orbital speed of 8 kilometers per second, they circle the planet in about 90 minutes. Some of the very low Earth orbits are not indefinitely stable because, as Earth's atmosphere swells from time to time, a frictional drag is generated by the atmosphere on these satellites, eventually leading to a loss of energy and "decay" of the orbit.

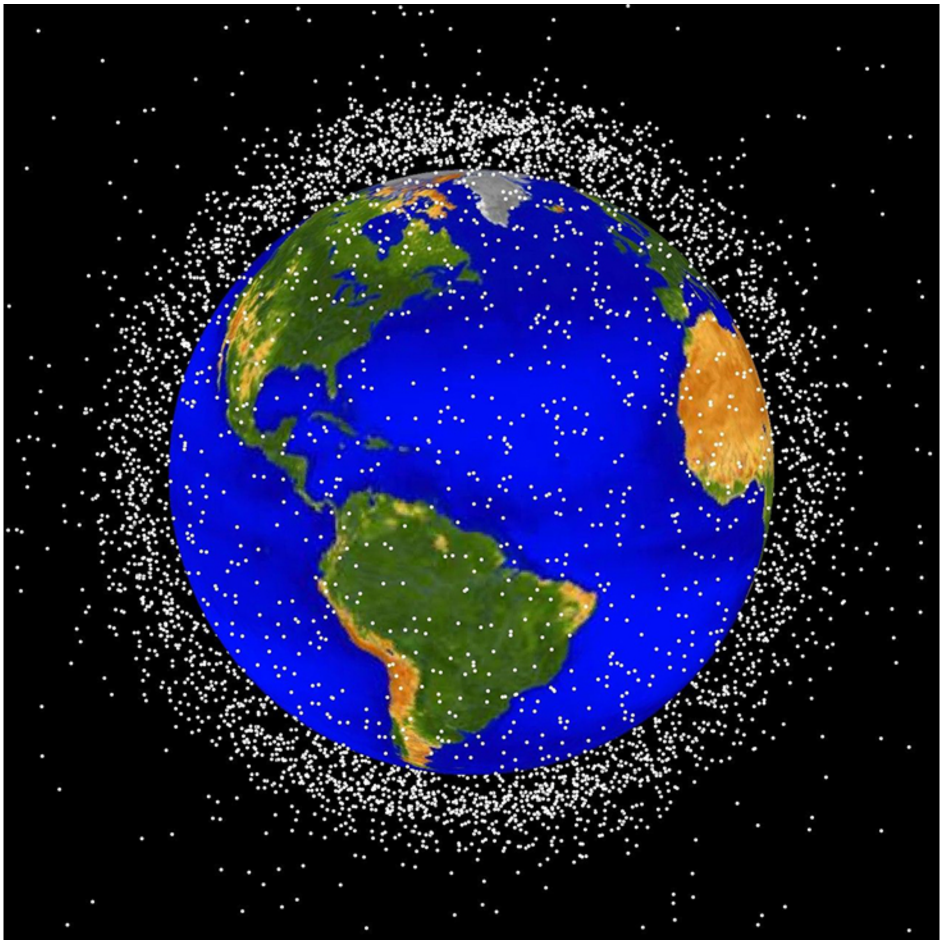


Figure 3.12 Satellites in Earth Orbit. This figure shows the larger pieces of orbital debris that are being tracked by NASA in Earth's orbit. (credit: NASA/JSC)

Interplanetary Spacecraft

The exploration of the solar system has been carried out largely by robot spacecraft sent to the other planets. To escape Earth, these craft must achieve **escape speed**, the speed needed to move away from Earth forever, which is about 11 kilometers per second (about 25,000 miles per hour). After escaping Earth, these craft coast to their targets, subject only to minor trajectory adjustments provided by small thruster rockets on board. In interplanetary flight, these spacecraft follow orbits around the Sun that are modified only when they pass near one of the planets.

As it comes close to its target, a spacecraft is deflected by the planet's gravitational force into a modified orbit, either gaining or losing energy in the process. Spacecraft controllers have actually been able to use a planet's gravity to redirect a flyby spacecraft to a second target. For example, Voyager 2 used a series of gravity-assisted encounters to yield successive flybys of Jupiter (1979), Saturn (1980), Uranus (1986), and Neptune (1989). The Galileo spacecraft, launched in 1989, flew past Venus once and Earth twice to gain the energy required to reach its ultimate goal of orbiting Jupiter.

If we wish to orbit a planet, we must slow the spacecraft with a rocket when the spacecraft is near its destination, allowing it to be captured into an elliptical orbit. Additional rocket thrust is required to bring a vehicle down from orbit for a landing on the surface. Finally, if a return trip to Earth is planned, the landed payload must include enough propulsive power to repeat the entire process in reverse.

3.6 GRAVITY WITH MORE THAN TWO BODIES

Learning Objectives

By the end of this section, you will be able to:

- Explain how the gravitational interactions of many bodies can causes perturbations in their motions
- Explain how the planet Neptune was discovered

Until now, we have considered the Sun and a planet (or a planet and one of its moons) as nothing more than a pair of bodies revolving around each other. In fact, all the planets exert gravitational forces upon one another as well. These interplanetary attractions cause slight variations from the orbits than would be expected if the gravitational forces between planets were neglected. The motion of a body that is under the gravitational influence of two or more other bodies is very complicated and can be calculated properly only with large computers. Fortunately, astronomers have such computers at their disposal in universities and government research institutes.

The Interactions of Many Bodies

As an example, suppose you have a cluster of a thousand stars all orbiting a common center (such clusters are quite common, as we shall see in **Star Clusters**). If we know the exact position of each star at any given instant, we can calculate the combined gravitational force of the entire group on any one member of the cluster. Knowing the force on the star in question, we can therefore find how it will accelerate. If we know how it was moving to begin with, we can then calculate how it will move in the next instant of time, thus tracking its motion.

However, the problem is complicated by the fact that the other stars are also moving and thus changing the effect they will have on our star. Therefore, we must simultaneously calculate the acceleration of each star produced by the combination of the gravitational attractions of all the others in order to track the motions of all of them, and hence of any one. Such complex calculations have been carried out with modern computers to track the evolution of hypothetical clusters of stars with up to a million members (**Figure 3.13**).

Figure 3.13 Modern Computing Power. These supercomputers at NASA's Ames Research Center are capable of tracking the motions of more than a million objects under their mutual gravitation. (credit: NASA Ames Research Center/Tom Trower)

Within the solar system, the problem of computing the orbits of planets and spacecraft is somewhat simpler. We have seen that Kepler's laws, which do not take into account the gravitational effects of the other planets on an orbit, really work quite well. This is because these additional influences are very small in comparison with the dominant gravitational attraction of the Sun. Under such circumstances, it is possible to treat the effects of other bodies as small **perturbations** (or disturbances). During the eighteenth and nineteenth centuries, mathematicians developed many elegant techniques for calculating perturbations, permitting them to predict very precisely the positions of the planets. Such calculations eventually led to the prediction and discovery of a new planet in 1846.

The Discovery of Neptune

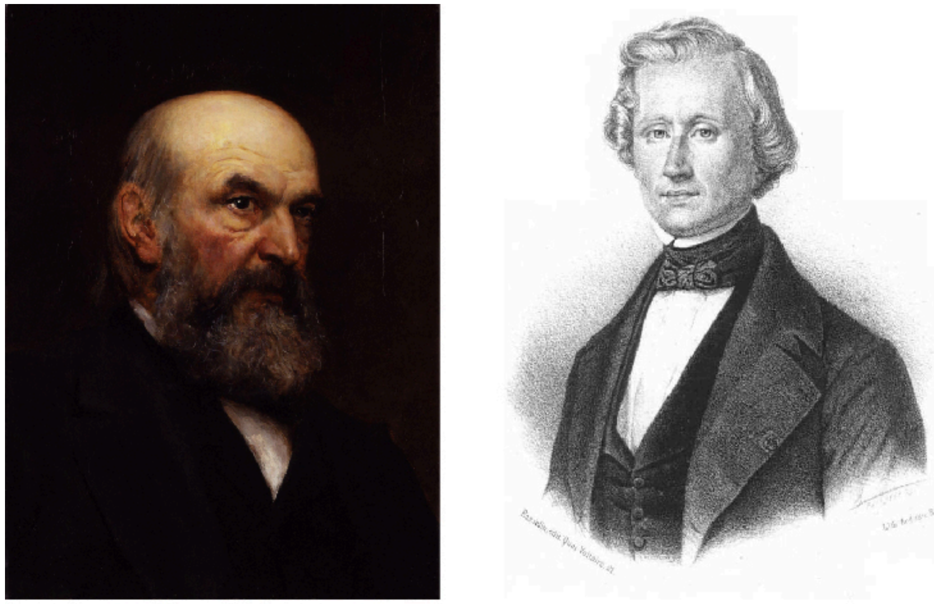
The discovery of the eighth planet, Neptune, was one of the high points in the development of gravitational theory. In 1781, William Herschel, a musician and amateur astronomer, accidentally discovered the seventh planet, Uranus. It happens that Uranus had been observed a century before, but in none of those earlier sightings was it recognized as a planet; rather, it was simply recorded as a star. Herschel’s discovery showed that there could be planets in the solar system too dim to be visible to the unaided eye, but ready to be discovered with a telescope if we just knew where to look.

By 1790, an orbit had been calculated for Uranus using observations of its motion in the decade following its discovery. Even after allowance was made for the perturbing effects of Jupiter and Saturn, however, it was found that Uranus did not move on an orbit that exactly fit the earlier observations of it made since 1690. By 1840, the discrepancy between the positions observed for Uranus and those predicted from its computed orbit amounted to about 0.03°—an angle barely discernable to the unaided eye but still larger than the probable errors in the orbital calculations. In other words, Uranus just did not seem to move on the orbit predicted from Newtonian theory.

In 1843, John Couch Adams, a young Englishman who had just completed his studies at Cambridge, began a detailed mathematical analysis of the irregularities in the motion of Uranus to see whether they might be produced by the pull of an unknown planet. He hypothesized a planet more distant from the Sun than Uranus, and then determined the mass and orbit it had to have to account for the departures in Uranus’ orbit. In October 1845, Adams delivered his results to George Airy, the British Astronomer Royal, informing him where in the sky to find the new planet. We now know that Adams’ predicted position for the new body was correct to within 2°, but for a variety of reasons, Airy did not follow up right away.

Meanwhile, French mathematician Urbain Jean Joseph Le Verrier, unaware of Adams or his work, attacked the same problem and published its solution in June 1846. Airy, noting that Le Verrier’s predicted position for the unknown planet agreed to within 1° with that of Adams, suggested to James Challis, Director of the Cambridge Observatory, that he begin a search for the new object. The Cambridge astronomer, having no up-to-date star charts of the Aquarius region of the sky where the planet was predicted to be, proceeded by recording the positions of all the faint stars he could observe with his telescope in that location. It was Challis’ plan to repeat such plots at intervals of several days, in the hope that the planet would distinguish itself from a star by its motion. Unfortunately, he was negligent in examining his observations; although he had actually seen the planet, he did not recognize it.

About a month later, Le Verrier suggested to Johann Galle, an astronomer at the Berlin Observatory, that he look for the planet. Galle received Le Verrier’s letter on September 23, 1846, and, possessing new charts of the Aquarius region, found and identified the planet that very night. It was less than a degree from the position Le Verrier predicted. The discovery of the eighth planet, now known as Neptune (the Latin name for the god of the sea), was a major triumph for gravitational theory for it dramatically confirmed the generality of Newton’s laws. The honor for the discovery is properly shared by the two mathematicians, Adams and Le Verrier (**Figure 3.14**).



(a) (b)

Figure 3.14 Mathematicians Who Discovered a Planet. (a) John Couch Adams (1819–1892) and (b) Urbain J. J. Le Verrier (1811–1877) share the credit for discovering the planet Neptune.

We should note that the discovery of Neptune was not a complete surprise to astronomers, who had long suspected the existence of the planet based on the “disobedient” motion of Uranus. On September 10, 1846, two weeks before Neptune was actually found, John Herschel, son of the discoverer of Uranus, remarked in a speech before the British Association, “We see [the new planet] as Columbus saw America from the shores of Spain. Its movements have been felt trembling along the far-reaching line of our analysis with a certainty hardly inferior to ocular demonstration.”

This discovery was a major step forward in combining Newtonian theory with painstaking observations. Such work continues in our own times with the discovery of planets around other stars.

LINK TO LEARNING

For the fuller story of how Neptune was predicted and found (and the effect of the discovery on the search for Pluto), you can read **this page (https://openstaxcollege.org/l/30nepplumatdis)** on the mathematical discovery of planets.

MAKING CONNECTIONS

Astronomy and the Poets

When Copernicus, Kepler, Galileo, and Newton formulated the fundamental rules that underlie everything in the physical world, they changed much more than the face of science. For some, they gave humanity the courage to let go of old superstitions and see the world as rational and manageable; for others, they upset comforting, ordered ways that had served humanity for centuries, leaving only a dry, “mechanical clockwork” universe in their wake.

Poets of the time reacted to such changes in their work and debated whether the new world picture was an appealing or frightening one. John Donne (1573–1631), in a poem called “Anatomy of the World,” laments the passing of the old certainties:

> The new Philosophy [science] calls all in doubt,
> The element of fire is quite put out;
> The Sun is lost, and th’ earth, and no man’s wit
> Can well direct him where to look for it.

(Here the “element of fire” refers also to the sphere of fire, which medieval thought placed between Earth and the Moon.)

By the next century, however, poets like Alexander Pope were celebrating Newton and the Newtonian world view. Pope’s famous couplet, written upon Newton’s death, goes

> Nature, and nature’s laws lay hid in night.
> God said, Let Newton be! And all was light.

In his 1733 poem, *An Essay on Man*, Pope delights in the complexity of the new views of the world, incomplete though they are:

> Of man, what see we, but his station here,
> From which to reason, to which refer? . . .
> He, who thro’ vast immensity can pierce,
> See worlds on worlds compose one universe,
> Observe how system into system runs,
> What other planets circle other suns,
> What vary’d being peoples every star,
> May tell why Heav’n has made us as we are . . .
> All nature is but art, unknown to thee;
> All chance, direction, which thou canst not see;
> All discord, harmony not understood;
> All partial evil, universal good:
> And, in spite of pride, in erring reason’s spite,
> One truth is clear, whatever is, is right.

Poets and philosophers continued to debate whether humanity was exalted or debased by the new views of science. The nineteenth-century poet Arthur Hugh Clough (1819–1861) cries out in his poem “The New Sinai”:

> And as old from Sinai’s top God said that God is one,
> By science strict so speaks He now to tell us, there is None!
> Earth goes by chemic forces; Heaven’s a Mécanique Celeste!
> And heart and mind of humankind a watchwork as the rest!

(A “mécanique celeste” is a clockwork model to demonstrate celestial motions.)

The twentieth-century poet Robinson Jeffers (whose brother was an astronomer) saw it differently in a poem called “Star Swirls”:

> There is nothing like astronomy to pull the stuff out of man.
> His stupid dreams and red-rooster importance:
> Let him count the star-swirls.

CHAPTER 3 REVIEW

KEY TERMS

angular momentum the measure of the motion of a rotating object in terms of its speed and how widely the object's mass is distributed around its axis

aphelion the point in its orbit where a planet (or other orbiting object) is farthest from the Sun

apogee the point in its orbit where an Earth satellite is farthest from Earth

asteroid belt the region of the solar system between the orbits of Mars and Jupiter in which most asteroids are located; the main belt, where the orbits are generally the most stable, extends from 2.2 to 3.3 AU from the Sun

astronomical unit (AU) the unit of length defined as the average distance between Earth and the Sun; this distance is about 1.5×10^8 kilometers

density the ratio of the mass of an object to its volume

eccentricity in an ellipse, the ratio of the distance between the foci to the major axis

ellipse a closed curve for which the sum of the distances from any point on the ellipse to two points inside (called the foci) is always the same

escape speed the speed a body must achieve to break away from the gravity of another body

focus (plural: foci) one of two fixed points inside an ellipse from which the sum of the distances to any point on the ellipse is constant

gravity the mutual attraction of material bodies or particles

Kepler's first law each planet moves around the Sun in an orbit that is an ellipse, with the Sun at one focus of the ellipse

Kepler's second law the straight line joining a planet and the Sun sweeps out equal areas in space in equal intervals of time

Kepler's third law the square of a planet's orbital period is directly proportional to the cube of the semimajor axis of its orbit

major axis the maximum diameter of an ellipse

momentum the measure of the amount of motion of a body; the momentum of a body is the product of its mass and velocity; in the absence of an unbalanced force, momentum is conserved

Newton's first law every object will continue to be in a state of rest or move at a constant speed in a straight line unless it is compelled to change by an outside force

Newton's second law the change of motion of a body is proportional to and in the direction of the force acting on it

Newton's third law for every action there is an equal and opposite reaction (*or:* the mutual actions of two bodies upon each other are always equal and act in opposite directions)

orbit the path of an object that is in revolution about another object or point

orbital period (*P*) the time it takes an object to travel once around the Sun

orbital speed the speed at which an object (usually a planet) orbits around the mass of another object; in the case of a planet, the speed at which each planet moves along its ellipse

perigee the point in its orbit where an Earth satellite is closest to Earth

perihelion the point in its orbit where a planet (or other orbiting object) is nearest to the Sun

perturbation a small disturbing effect on the motion or orbit of a body produced by a third body

satellite an object that revolves around a planet

semimajor axis half of the major axis of a conic section, such as an ellipse

velocity the speed and direction a body is moving—for example, 44 kilometers per second toward the north galactic pole

SUMMARY

3.1 The Laws of Planetary Motion

Tycho Brahe's accurate observations of planetary positions provided the data used by Johannes Kepler to derive his three fundamental laws of planetary motion. Kepler's laws describe the behavior of planets in their orbits as follows: (1) planetary orbits are ellipses with the Sun at one focus; (2) in equal intervals, a planet's orbit sweeps out equal areas; and (3) the relationship between the orbital period (*P*) and the semimajor axis (*a*) of an orbit is given by $P^2 = a^3$ (when *a* is in units of AU and *P* is in units of Earth years).

3.2 Newton's Great Synthesis

In his *Principia*, Isaac Newton established the three laws that govern the motion of objects: (1) objects continue to be at rest or move with a constant velocity unless acted upon by an outside force; (2) an outside force causes an acceleration (and changes the momentum) for an object; and (3) for every action there is an equal and opposite reaction. Momentum is a measure of the motion of an object and depends on both its mass and its velocity. Angular momentum is a measure of the motion of a spinning or revolving object and depends on its mass, velocity, and distance from the point around which it revolves. The density of an object is its mass divided by its volume.

3.3 Newton's Universal Law of Gravitation

Gravity, the attractive force between all masses, is what keeps the planets in orbit. Newton's universal law of gravitation relates the gravitational force to mass and distance:

$$F_{\text{gravity}} = G\frac{M_1 M_2}{R^2}$$

The force of gravity is what gives us our sense of weight. Unlike mass, which is constant, weight can vary depending on the force of gravity (or acceleration) you feel. When Kepler's laws are reexamined in the light of Newton's gravitational law, it becomes clear that the masses of both objects are important for the third law, which becomes $a^3 = (M_1 + M_2) \times P^2$. Mutual gravitational effects permit us to calculate the masses of astronomical objects, from comets to galaxies.

3.4 Orbits in the Solar System

The closest point in a satellite orbit around Earth is its perigee, and the farthest point is its apogee (corresponding to perihelion and aphelion for an orbit around the Sun). The planets follow orbits around the Sun that are nearly circular and in the same plane. Most asteroids are found between Mars and Jupiter in the asteroid belt, whereas comets generally follow orbits of high eccentricity.

3.5 Motions of Satellites and Spacecraft

The orbit of an artificial satellite depends on the circumstances of its launch. The circular satellite velocity needed to orbit Earth's surface is 8 kilometers per second, and the escape speed from our planet is 11 kilometers per second. There are many possible interplanetary trajectories, including those that use gravity-assisted flybys of one object to redirect the spacecraft toward its next target.

3.6 Gravity with More Than Two Bodies

Calculating the gravitational interaction of more than two objects is complicated and requires large computers. If one object (like the Sun in our solar system) dominates gravitationally, it is possible to calculate the effects of a second object in terms of small perturbations. This approach was used by John Couch Adams and Urbain Le Verrier to predict the position of Neptune from its perturbations of the orbit of Uranus and thus discover a new planet mathematically.

FOR FURTHER EXPLORATION

Articles

Brahe and Kepler

Christianson, G. "The Celestial Palace of Tycho Brahe." *Scientific American* (February 1961): 118.

Gingerich, O. "Johannes Kepler and the Rudolphine Tables." *Sky & Telescope* (December 1971): 328. Brief article on Kepler's work.

Wilson, C. "How Did Kepler Discover His First Two Laws?" *Scientific American* (March 1972): 92.

Newton

Christianson, G. "Newton's *Principia*: A Retrospective." *Sky & Telescope* (July 1987): 18.

Cohen, I. "Newton's Discovery of Gravity." *Scientific American* (March 1981): 166.

Gingerich, O. "Newton, Halley, and the Comet." *Sky & Telescope* (March 1986): 230.

Sullivant, R. "When the Apple Falls." *Astronomy* (April 1998): 55. Brief overview.

The Discovery of Neptune

Sheehan, W., et al. "The Case of the Pilfered Planet: Did the British Steal Neptune?" *Scientific American* (December 2004): 92.

Websites

Brahe and Kepler

Johannes Kepler: His Life, His Laws, and Time: **http://kepler.nasa.gov/Mission/JohannesKepler/ (http://kepler.nasa.gov/Mission/JohannesKepler/)** . From NASA's Kepler mission.

Johannes Kepler: **http://www.britannica.com/biography/Johannes-Kepler (http://www.britannica.com/**

biography/Johannes-Kepler) . Encyclopedia Britannica article.

Johannes Kepler: http://www-history.mcs.st-andrews.ac.uk/Biographies/Kepler.html (http://www-history.mcs.st-andrews.ac.uk/Biographies/Kepler.html) . MacTutor article with additional links.

Noble Dane: Images of Tycho Brahe: http://www.mhs.ox.ac.uk/tycho/index.htm (http://www.mhs.ox.ac.uk/tycho/index.htm) . A virtual museum exhibit from Oxford.

Newton

Sir Isaac Newton: http://www-groups.dcs.st-and.ac.uk/~history//Biographies/Newton.html (http://www-groups.dcs.st-and.ac.uk/~history//Biographies/Newton.html) . MacTutor article with additional links.

Sir Isaac Newton: http://www.luminarium.org/sevenlit/newton/newtonbio.htm (http://www.luminarium.org/sevenlit/newton/newtonbio.htm) . Newton Biography at the Luminarium.

The Discovery of Neptune

Adams, Airy, and the Discovery of Neptune: http://www.mikeoates.org/lassell/adams-airy.htm (http://www.mikeoates.org/lassell/adams-airy.htm) . A defense of Airy's role by historian Alan Chapman.

Mathematical Discovery of Planets: http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Neptune_and_Pluto.html (http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Neptune_and_Pluto.html) . MacTutor article.

Videos

Brahe and Kepler

"Harmony of the Worlds." This third episode of Carl Sagan's TV series *Cosmos* focuses on Kepler and his life and work.

Tycho Brahe, Johannes Kepler, and Planetary Motion: https://www.youtube.com/watch?v=x3ALuycrCwI (https://www.youtube.com/watch?v=x3ALuycrCwI) . German-produced video, in English (14:27).

Newton

Beyond the Big Bang: Sir Isaac Newton's Law of Gravity: http://www.history.com/topics/enlightenment/videos/beyond-the-big-bang-sir-isaac-newtons-law-of-gravity (http://www.history.com/topics/enlightenment/videos/beyond-the-big-bang-sir-isaac-newtons-law-of-gravity) . From the History Channel (4:35).

Sir Isaac Newton versus Bill Nye: Epic Rap Battles of History: https://www.youtube.com/watch?v=8yis7GzlXNM (https://www.youtube.com/watch?v=8yis7GzlXNM) . (2:47).

The Discovery of Neptune

Richard Feynman: On the Discovery of Neptune: https://www.youtube.com/watch?v=FgXQffVgZRs (https://www.youtube.com/watch?v=FgXQffVgZRs) . A brief black-and-white Caltech lecture (4:33).

COLLABORATIVE GROUP ACTIVITIES

A. An eccentric, but very rich, alumnus of your college makes a bet with the dean that if you drop a baseball and a bowling ball from the tallest building on campus, the bowling ball would hit the ground first. Have your group discuss whether you would make a side bet that the alumnus is right. How would you decide who is right?

B. Suppose someone in your astronomy class was unhappy about his or her weight. Where could a person go to weigh one-fourth as much as he or she does now? Would changing the unhappy person's weight have any effect on his or her mass?

C. When the Apollo astronauts landed on the Moon, some commentators commented that it ruined the mystery and "poetry" of the Moon forever (and that lovers could never gaze at the full moon in the same way again). Others felt that knowing more about the Moon could only enhance its interest to us as we see it from Earth. How do the various members of your group feel? Why?

D. **Figure 3.12** shows a swarm of satellites in orbit around Earth. What do you think all these satellites do? How many categories of functions for Earth satellites can your group come up with?

E. The Making Connections feature box **Astronomy and the Poets** discusses how poets included the most recent astronomical knowledge in their poetry. Is this still happening today? Can your group members come up with any poems or songs that you know that deal with astronomy or outer space? If not, perhaps you could find some online, or by asking friends or roommates who are into poetry or music.

EXERCISES

Review Questions

1. State Kepler's three laws in your own words.

2. Why did Kepler need Tycho Brahe's data to formulate his laws?

3. Which has more mass: an armful of feathers or an armful of lead? Which has more volume: a kilogram of feathers or a kilogram of lead? Which has higher density: a kilogram of feathers or a kilogram of lead?

4. Explain how Kepler was able to find a relationship (his third law) between the orbital periods and distances of the planets that did not depend on the masses of the planets or the Sun.

5. Write out Newton's three laws of motion in terms of what happens with the momentum of objects.

6. Which major planet has the largest . . .

- **A.** semimajor axis?
- **B.** average orbital speed around the Sun?
- **C.** orbital period around the Sun?
- **D.** eccentricity?

7. Why do we say that Neptune was the first planet to be discovered through the use of mathematics?

8. Why was Brahe reluctant to provide Kepler with all his data at one time?

9. According to Kepler's second law, where in a planet's orbit would it be moving fastest? Where would it be moving slowest?

10. The gas pedal, the brakes, and the steering wheel all have the ability to accelerate a car—how?

11. Explain how a rocket can propel itself using Newton's third law.

12. A certain material has a mass of 565 g while occupying 50 cm^3 of space. What is this material? (Hint: Use **Table 3.1**.)

13. To calculate the momentum of an object, which properties of an object do you need to know?

14. To calculate the angular momentum of an object, which properties of an object do you need to know?

15. What was the great insight Newton had regarding Earth's gravity that allowed him to develop the universal law of gravitation?

16. Which of these properties of an object best quantifies its inertia: velocity, acceleration, volume, mass, or temperature?

17. Pluto's orbit is more eccentric than any of the major planets. What does that mean?

18. Why is Tycho Brahe often called "the greatest naked-eye astronomer" of all time?

Thought Questions

19. Is it possible to escape the force of gravity by going into orbit around Earth? How does the force of gravity in the International Space Station (orbiting an average of 400 km above Earth's surface) compare with that on the ground?

20. What is the momentum of an object whose velocity is zero? How does Newton's first law of motion include the case of an object at rest?

21. Evil space aliens drop you and your fellow astronomy student 1 km apart out in space, very far from any star or planet. Discuss the effects of gravity on each of you.

22. A body moves in a perfectly circular path at constant speed. Are there forces acting in such a system? How do you know?

23. As friction with our atmosphere causes a satellite to spiral inward, closer to Earth, its orbital speed increases. Why?

24. Use a history book, an encyclopedia, or the internet to find out what else was happening in England during Newton's lifetime and discuss what trends of the time might have contributed to his accomplishments and the rapid acceptance of his work.

25. Two asteroids begin to gravitationally attract one another. If one asteroid has twice the mass of the other, which one experiences the greater force? Which one experiences the greater acceleration?

26. How does the mass of an astronaut change when she travels from Earth to the Moon? How does her weight change?

27. If there is gravity where the International Space Station (ISS) is located above Earth, why doesn't the space station get pulled back down to Earth?

28. Compare the density, weight, mass, and volume of a pound of gold to a pound of iron on the surface of Earth.

29. If identical spacecraft were orbiting Mars and Earth at identical radii (distances), which spacecraft would be moving faster? Why?

Figuring For Yourself

30. By what factor would a person's weight be increased if Earth had 10 times its present mass, but the same volume?

31. Suppose astronomers find an earthlike planet that is twice the size of Earth (that is, its radius is twice that of Earth's). What must be the mass of this planet such that the gravitational force (F_{gravity}) at the surface would be identical to Earth's?

32. What is the semimajor axis of a circle of diameter 24 cm? What is its eccentricity?

33. If 24 g of material fills a cube 2 cm on a side, what is the density of the material?

34. If 128 g of material is in the shape of a brick 2 cm wide, 4 cm high, and 8 cm long, what is the density of the material?

35. If the major axis of an ellipse is 16 cm, what is the semimajor axis? If the eccentricity is 0.8, would this ellipse be best described as mostly circular or very elongated?

36. What is the average distance from the Sun (in astronomical units) of an asteroid with an orbital period of 8 years?

37. What is the average distance from the Sun (in astronomical units) of a planet with an orbital period of 45.66 years?

38. In 1996, astronomers discovered an icy object beyond Pluto that was given the designation 1996 TL 66. It has a semimajor axis of 84 AU. What is its orbital period according to Kepler's third law?