Part 5: Magnetic Forces and Fields

University Physics V2 (Openstax): Chapters 11, 12 & 13
Physics for Engineers & Scientists (Giancoli): Chapters 27, 28 & 29

Magnetic Forces and Fields

- Magnetic forces are similar to electric forces, except there are NO magnetic charges.
- "Magnetic Monopoles", theoretical magnetic charges, have never been observed.
- The ends of magnets act like charges (North & South)
- Like poles repel, opposite poles attract
- Magnetic poles create magnetic fields
- Field lines move from "north" poles to "south" poles
- These forces and fields, actually come from atoms themselves, which act as tiny magnets.

Magnetic Forces

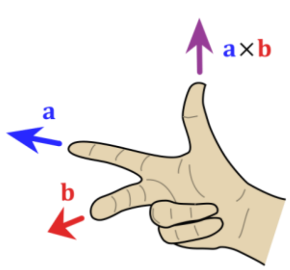
$$\vec{F} = q\vec{v} \times \vec{B} \qquad |\vec{F}| = |q\vec{v} \times \vec{B}| = |q\vec{v}||\vec{B}|\,Sin\theta$$

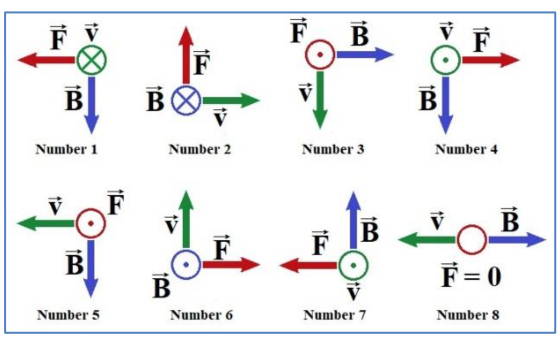
$$\vec{F} = q\det\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} = q(v_yB_z - v_zB_y)\hat{i} + q(v_zB_x - v_xB_z)\hat{j} + q(v_xB_y - v_yB_x)\hat{k}$$

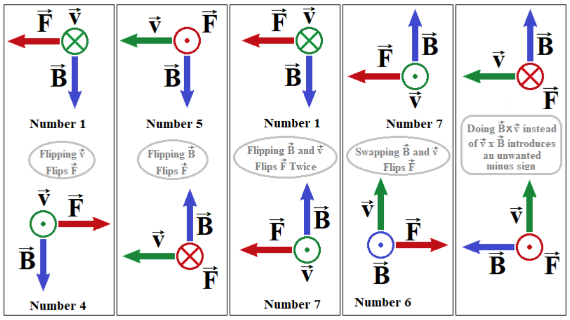
- F = Force felt by charge
- q = charge
- v = velocity of the charge
- B = Magnetic field

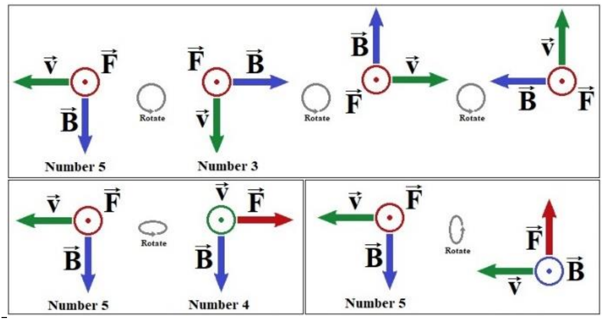
- Only moving charges are affected by magnetic forces.
- The velocity must have a component ⊥ to the magnetic field.
- The units of the magnetic field are the Tesla: 1T = 1 N·s/(C·m).

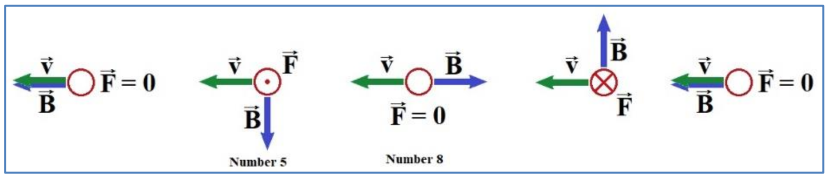
Right Hand Rule



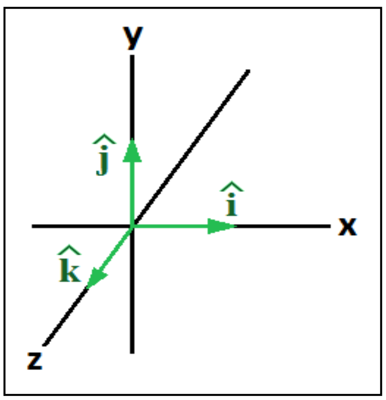








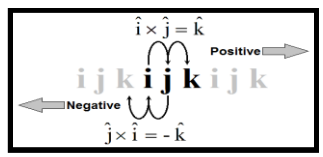
Unit Vectors



$\hat{i}\times\hat{i} = 0 \qquad \hat{j}\times\hat{j} = 0 \qquad \hat{k}\times\hat{k} = 0$

$\hat{i}\times\hat{j} = \hat{k} \qquad \hat{i}\times\hat{k} = -\hat{j} \qquad \hat{j}\times\hat{k} = \hat{i}$

$\hat{j}\times\hat{i} = -\hat{k} \qquad \hat{k}\times\hat{i} = \hat{j} \qquad \hat{k}\times\hat{j} = -\hat{i}$



Example: A particle with a charge of +8.40 μC and a speed of 45.0 m/s enters a uniform magnetic field whose magnitude is 0.300T. What is the force on the charge if the angle between its velocity and the magnetic field is 30.0°?

$$|\vec{F}| = |q\vec{v}||\vec{B}|\,Sin\theta = (8.40\mu C)(45.0\text{ m/s})(0.300T)Sin30° = 56.7\mu N$$

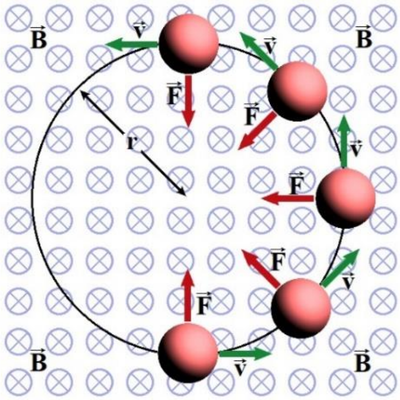
Example: A velocity of a +17.6 μC charge is v = (24.0m/s) î + (15.0 m/s)ĵ . It is moving in a region with a uniform magnetic field B = (0.200 T) î + (0.250T)ĵ. Determine the force felt by the charge.

$$\vec{F} = q\vec{v}\times\vec{B} = (17.6\mu C)[(24.0\text{m/s})\hat{i} + (15.0\text{m/s})\hat{j}]\times[(0.200T)\hat{i} + (0.250T)\hat{j}]$$

$$\vec{F} = (17.6\mu C)[(24.0\text{m/s})(0.200T)(\hat{i}\times\hat{i}) + (24.0\text{m/s})(0.250T)(\hat{i}\times\hat{j}) + (15.0\text{m/s})(0.200T)(\hat{j}\times\hat{i}) + (15.0\text{m/s})(0.250T)(\hat{j}\times\hat{j})]$$

$$\vec{F} = (17.6\mu C)[(24.0\text{m/s})(0.250T)(\hat{k}) + (15.0\text{m/s})(0.200T)(-\hat{k})] = (52.8\mu N)\hat{k}$$

Charged Particle Motion in a Magnetic Field



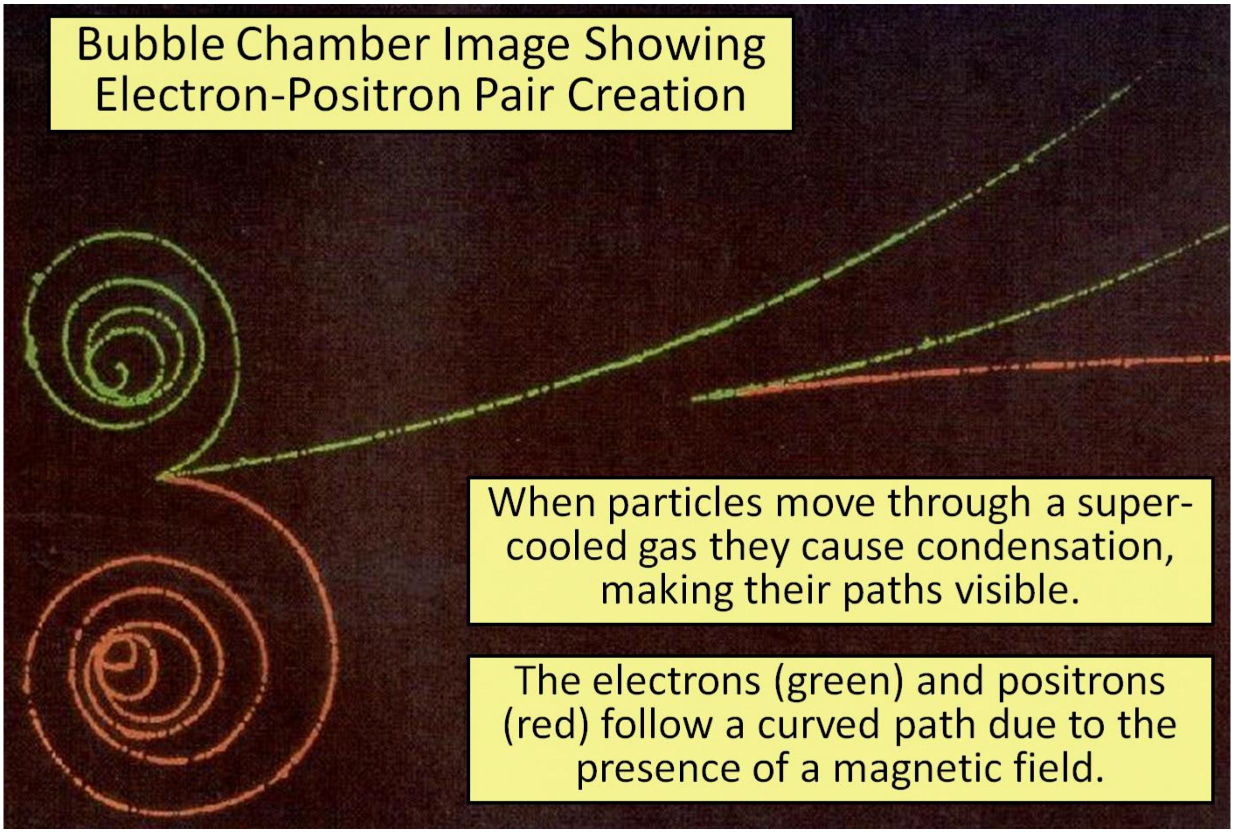
- F is always ⊥ to v → changes direction, not speed
- Uniform circular motion can occur.

$$F = qvB = F_C = \frac{mv^2}{r} \qquad r = \frac{mv}{qB}$$

Example: A charged particle with a charge-to-mass ratio of q/m = 5.70 x 10^8 C/kg travels on a circular path that is perpendicular to a magnetic field of magnitude 0.720 T. How much time does it take the particle to make one complete revolution?

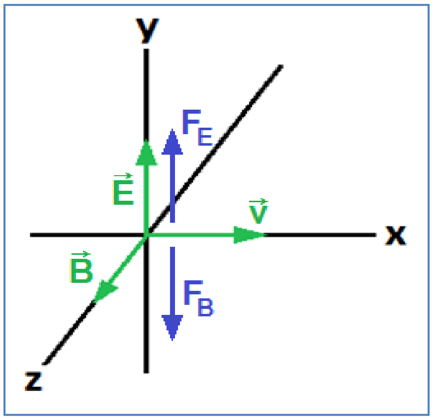
$$v = \frac{dist}{time} = \frac{2\pi r}{T} \qquad T = \frac{2\pi r}{v} \qquad\qquad r = \frac{mv}{qB} \qquad \frac{r}{v} = \frac{m}{qB}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB} = \frac{2\pi}{(q/m)B} = \frac{2\pi}{(5.70\times10^8 C/kg)(0.720T)} = 1.5\times10^{-8}\text{s}$$



Lorentz Equation: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

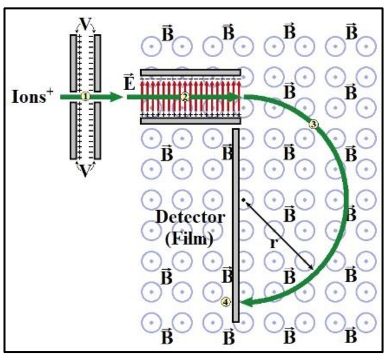
- A **Velocity Filter** can be made using "crossed fields"



$$\sum \vec{F} = qE - qvB = 0 \qquad v = \frac{E}{B}$$

The only charged particles that can pass undeflected are those with v=E/B

Mass Spectrometer



① Ions from source (sample) are accelerated through a potential V.

$$eV = \frac{1}{2}mv^2 \qquad v = \sqrt{\frac{2eV}{m}}$$

② Ions pass through "crossed fields", which act as a velocity selector.

$$F_E = eE = F_B = evB \qquad v = E/B$$

③ The ions then follow a curved path (due to B) until measured at the film. The radius of the path indicates the mass.

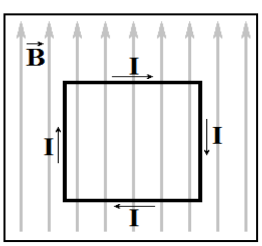
$$r = \frac{mv}{eB} \qquad m = \frac{reB}{v} \qquad m^2 = \frac{r^2e^2B^2}{v^2} = \frac{r^2e^2B^2m}{2eV} \qquad m = \frac{er^2B^2}{2V}$$

Force on a Current Carrying Wire $\vec{F} = I\vec{L} \times \vec{B}$

- In an infinitesimal amount of time Δt an infinitesimal amount of charge Δq passes by a spot in our wire.

$$F = \Delta q \cdot vBSin\theta = \left(\frac{\Delta q}{\Delta t}\right)(v \cdot \Delta t)BSin\theta = ILBSin\theta$$

Example: A square coil of wire containing a single turn is placed in a 0.250 T magnetic field as shown. Each side has a length of 32.0 cm and the current in the coil is 12.0 A. Determine the magnetic force on each of the 4 sides.



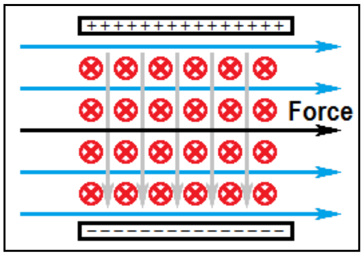
On right and left sides, L is parallel to B: F=0

On top and bottom, L ⊥ B:

F = ILB = (12.0A)(0.320m)(0.250T) = 0.960 N

F points outward on top and inward on bottom

Magnetohydrodynamic Propulsion



① Sea water flows through an intake into a chamber. An electrical potential is established on plates on opposite sides, creating an electric field in the chamber.

② Ion Ions in the sea water are pushed by the electric field, creating an electric current, which moves across the chamber.

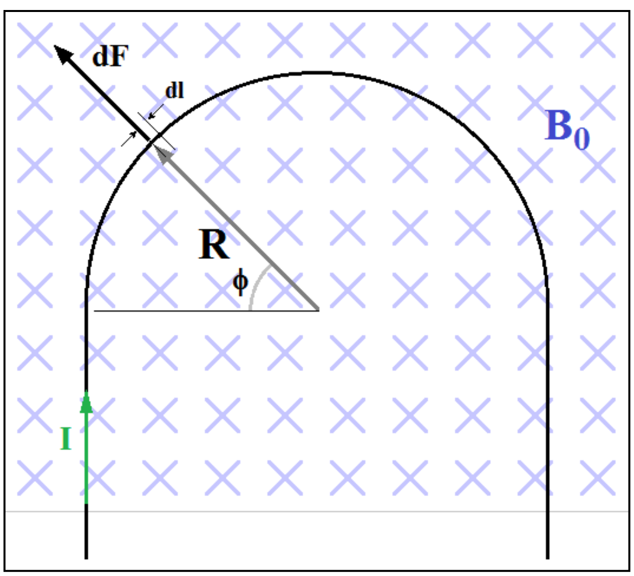
③ A magnetic field is added ⊥ to the electric field. The electric current interacts with the magnetic field, resulting in a force that pushes the charges through.

④ Pushing water out the back can be use to propel a vehicle forward (Newton’s 3rd Law) very quietly (as there are no moving parts).

Magnetic Forces From Varying Currents

- If I, L, B, or θ varies, use $d\vec{F} = Id\vec{L} \times \vec{B}$ (instead of $\vec{F} = I\vec{L} \times \vec{B}$) and integrate.

Example: A rigid wire carrying a current I, consists of a semicircle of radius R, and two straight portions as shown. The wire lies in a plane ⊥ to a uniform magnetic field of magnitude B_0. The straight portion of the wire have length=a within the field. Determine the net force on the wire.



Straight Sections: $F = ILBSinq = ILB_0$
These point in opposite directions (cancel)

Curved Sections: $dF = IBSinqdL = IB_0dL$

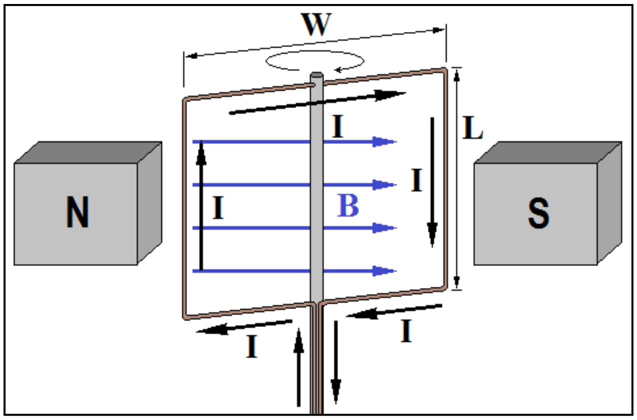
dF = vector → Must sum components
x-components will cancel (symmetry)

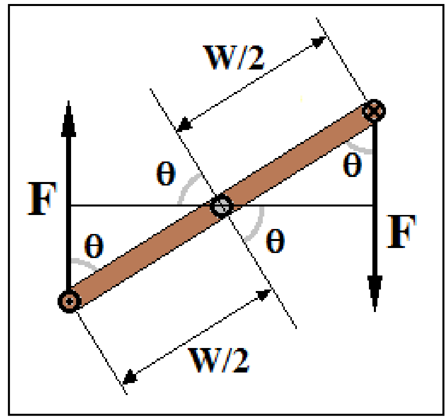
$$\left|\vec{F}\right| = F_y = \int_0^{\pi} dF \cdot Sin\phi = \int_0^{\pi} IB_0 \cdot Sin\phi \cdot dL$$

$$\left|\vec{F}\right| = \int_0^{\pi} IB_0 Sin\phi \cdot Rd\phi = IB_0 R \int_0^{\pi} Sin\phi d\phi$$

$$\left|\vec{F}\right| = IB_0 R\left[-Cos\phi\right]_0^{\pi} = 2IB_0 R$$

Torque on a Current Carrying Wire $\tau = NAIBSin(\theta)$





Torque from one arm: $\tau = FrSin\theta = (ILB)\left(\frac{W}{2}\right)Sin\theta$

Torque from both arms: $\tau = ILWBSin\theta = IABSin\theta$

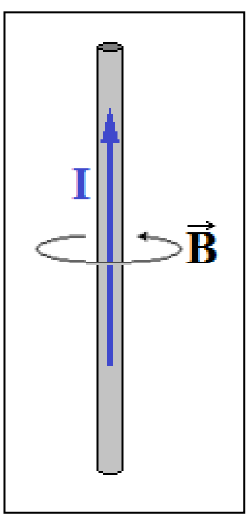
Torque from N turns: $\tau = NIABSin\theta$

Magnetic Dipole Moment: $\vec{\mu} = NI\vec{A}$ $\vec{\tau} = \vec{\mu} \times \mathbf{B}$

Example: A 1200-turn coil in a DC motor has an area per turn of $1.10 \times 10^{-2} m^2$. The design for the motor specifies that the magnitude of the maximum torque is 5.80 N·m when the coil is placed in a 0.200 T magnetic field. What is the current in the coil?

$\tau = NIABSin\theta$ $\tau_{max} = NIAB$ $I = \frac{\tau_{max}}{NAB} = \frac{(5.80\,N \cdot m)}{(1200)(1.10 \times 10^{-2} m^2)(0.200T)} = 2.2A$

Magnetic Fields Produced by Currents: $B = \frac{\mu_0 I}{2\pi r}$

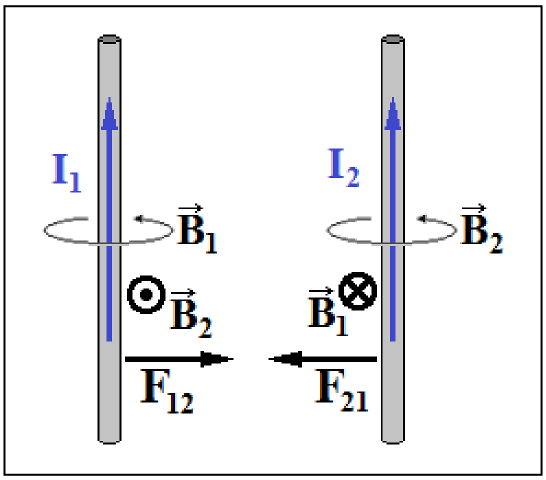


- B = Magnetic field
- I = Current
- r = Distance from wire
- μ_0 = "Permeability of free space" = $4\pi x10^{-7}$ T·m/A

Example: A long straight wire carries a current of 48.0 A. The magnetic field produced by this current at a certain point is 80.0 μT. How far is this point from the wire?

$$r = \frac{\mu_0 I}{2\pi B} = \left(\frac{\mu_0}{2\pi}\right)\frac{\mathrm{I}}{\mathrm{B}} = \left(2\times10^{-7}\,T\cdot m/A\right)\frac{48.0\mathrm{A}}{80.0\times10^{-6}\,T} = 0.120\mathrm{m}$$

Attraction/Repulsion of Two Current-Carrying Wires



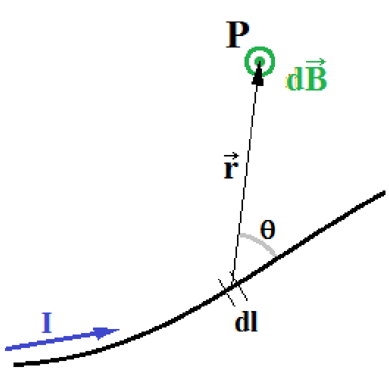
$$B_1 = \frac{\mu_0 I_1}{2\pi r} \qquad F_{21} = I_2 L B_1$$

$$B_2 = \frac{\mu_0 I_2}{2\pi r} \qquad F_{12} = I_1 L B_2$$

$$F = F_{21} = F_{12} = \frac{\mu_0 I_1 I_2 L}{2\pi r}$$

- Attractive or repulsive?
  - Currents in same direction = Attractive
  - Currents in opposite directions = Repulsive
- What if the wire on the right is rotated so the I_2 goes into or out of the page?
  - For both wires, B and I will be parallel: Sin θ = 0 F=ILBSinθ = 0

Biot-Savart Law: $d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\hat{r}}{r^2}$ $\quad \vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{l}\times\hat{r}}{r^2}$



Biot-Savart Law produces the magnetic field created by any wire (straight or curved)

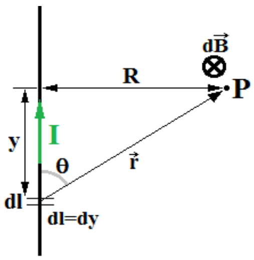
$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\hat{r}}{r^2} \qquad \vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{l}\times\hat{r}}{r^2}$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} \qquad r^2 = \vec{r}\cdot\vec{r} = |\vec{r}|^2$$

Note: This is a vector sum!

$$dB = \frac{\mu_0 I Sin\theta}{4\pi r^2}dl$$

Example: Biot-Savart Law on Long Straight Wire



1) Mark a "dl"
2) Draw in r vector & θ
3) Find variable for "dl"
4) Find direction of dB

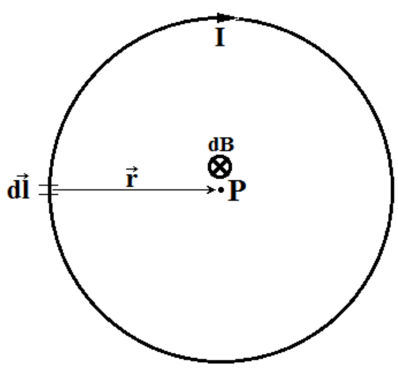
$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\hat{r}}{r^2} \qquad dB = \frac{\mu_0 I Sin\theta}{4\pi r^2}dy \qquad B = \frac{\mu_0 I}{4\pi}\int\frac{Sin\theta}{r^2}dy$$

$$Sin\theta = \frac{R}{r} \qquad B = \frac{\mu_0 I}{4\pi}\int\frac{R}{r^3}dy \qquad r = \sqrt{R^2 + y^2} \qquad B = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{R}{\left[R^2 + y^2\right]^{3/2}}dy$$

From Integral Table: $\int\left[x^2 + a^2\right]^{-3/2}dx = \frac{x}{a^2\sqrt{x^2 + a^2}}$

$$B = \frac{\mu_0 IR}{4\pi}\left[\frac{y}{R^2\sqrt{y^2 + R^2}}\right]_{-\infty}^{\infty} = \frac{\mu_0 IR}{4\pi}\left[\frac{1}{R^2} - \left(\frac{-1}{R^2}\right)\right] = \frac{\mu_0 I}{2\pi R}$$

Example: Biot-Savart Law to Find B at the Center of a Current Loop



$$\vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{l}\times\hat{r}}{r^2}$$

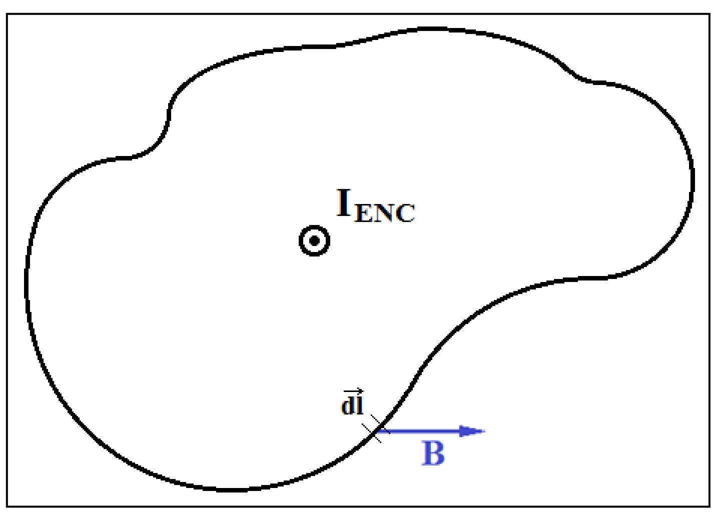
All of the dB's point the same direction

→ Integrate magnitude (no need to sum by components)

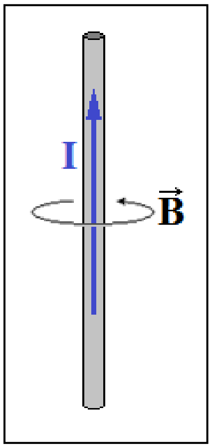
$$\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{Sin\theta}{r^2}dl$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{1}{r^2}dl = \frac{\mu_0 I}{4\pi r^2}\int dl = \frac{\mu_0 I}{4\pi r^2}(2\pi r) = \frac{\mu_0 I}{2r}$$

Ampere's Law: $\oint\vec{B}\cdot d\vec{l} = \mu_0 I_{ENC}$

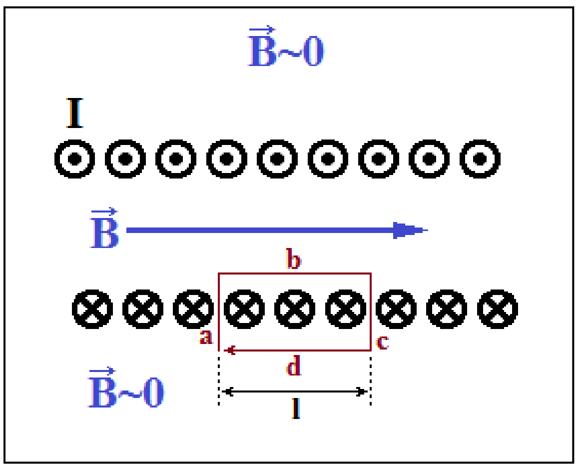


Ampere's Law on Long Straight Wire



- Due to symmetry B is constant around loop
- B is parallel to Δl
- $\oint \vec{B}\cdot d\vec{l} = \oint Bdl = B\oint dl = B(2\pi r)$
- $B(2\pi r) = \mu_0 I \qquad B = \dfrac{\mu_0 I}{2\pi r}$

Ampere's Law on Solenoid $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{ENC}$



- Draw in a loop…
- $\oint \vec{B}\cdot d\vec{l} = \int_a \vec{B}\cdot d\vec{l} + \int_b \vec{B}\cdot d\vec{l} + \int_c \vec{B}\cdot d\vec{l} + \int_d \vec{B}\cdot d\vec{l}$
- Paths a & c are ⊥ to B: $\int_a \vec{B}\cdot d\vec{l} = \int_c \vec{B}\cdot d\vec{l} = 0$
- Outside B≈0: $\int_d \vec{B}\cdot d\vec{l} = 0$
- $\oint \vec{B}\cdot d\vec{l} = \int_b \vec{B}\cdot d\vec{l} = \int_b Bdl = B\int_b dl = Bl$
- $Bl = \mu_0(NI) \quad B = \dfrac{\mu_0 NI}{l} = \mu_0 nI$
- N = Number of Turns   n = Turns/length

Example: What is the magnetic field produced in the center of a solenoid with 500.0 turns per meter carrying a current of 2.00A?

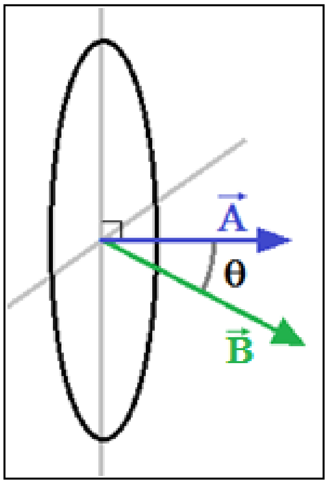
$$B = \mu_0 nI = (4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(500.0\,\mathrm{m^{-1}})(2.00\mathrm{A}) = 1.26\,\mathrm{mT}$$

Example: A 1250 turn solenoid that is 25.0 cm in length carries a current of 8.00mA. If an electron moves in a circular path in the center with a speed of 5.64x10^5 m/s, what is the radius of its path?

$$B = \frac{\mu_0 NI}{L} = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(1250)(8.00\times 10^{-3}\,\mathrm{A})}{0.25m} = 50.265\,\mu\mathrm{T}$$

$$r = \frac{mv}{qB} = \frac{(9.11\times 10^{-31}kg)(5.64\times 10^{5}\,m/s)}{(1.60\times 10^{-19}C)(50.265\times 10^{-6}T)} = 63.9mm$$

Magnetic Flux



$$\underbrace{\Phi_B}_{\substack{Magnetic\\Flux}} = \vec{B} \cdot \underbrace{\vec{A}}_{Area}$$

$$\vec{A} \perp Surface$$

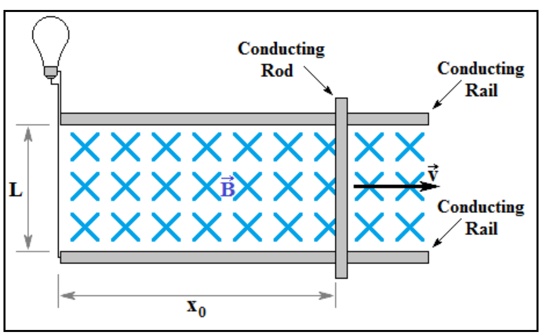
$$\left|\vec{A}\right| = Area$$

- Constant magnetic flux does nothing!
- Changing Magnetic flux (by changing B, A, or θ) creates an emf.

Faraday's Law (Electromagnetic Induction): $|emf| = N\dfrac{d\Phi_B}{dt}$

- $|emf| = N\dfrac{d(BACos\theta)}{dt} = N\dfrac{dB}{dt}ACos\theta + NB\dfrac{dA}{dt}Cos\theta + NBA\dfrac{d(Cos\theta)}{dt}$
- Currents and emfs created in this manner are called "induced"
- $|emf|_{AVG} = N\dfrac{\Delta\Phi_B}{\Delta t} = N\dfrac{(\Phi_B)_{Final} - (\Phi_B)_{Initial}}{\Delta t}$

Example: When the conducting rod of length L = 1.60m is pulled to the right at v = 5.00 m/s through the magnetic field B = 0.800 T in the figure below, the 96.0 Ω bulb lights. Determine (a) the average emf delivered to the bulb, (b) the current in the bulb, and (c) the power delivered.



B = constant q = constant

$$|emf| = NB\frac{A_{Final} - A_{init}}{\Delta t}Cos\theta$$

Cos q = 1 N = 1

$$A_{Final} - A_{init} = [L(x_0 + v\Delta t)] - [Lx_0] = Lv\Delta t$$

a) $|emf| = NB\dfrac{A_{Final} - A_{init}}{\Delta t}Cos\theta = B\dfrac{Lv\Delta t}{\Delta t} = BLv = (0.800T)(1.60m)(5.00m/s) = 6.40V$

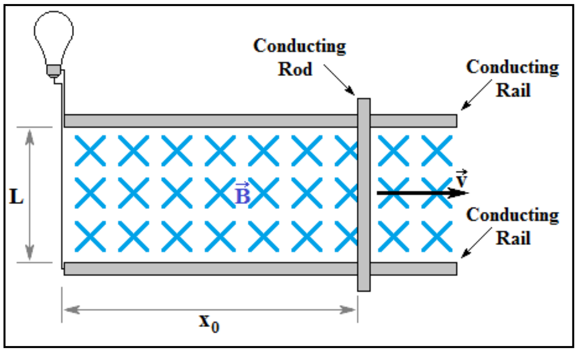
b) $I = \dfrac{emf}{R} = \dfrac{6.40V}{96.0\Omega} = 66.7mA$ c) $P = IV = (66.7mA)(6.4V) = 0.427W$

Where does this energy come from? $P = \dfrac{W}{t} = \dfrac{Fd}{t} = Fv = (ILB)v = I(BLv) = IV$

Lenz's Law : An induced emf resulting from changing magnetic flux has a polarity that leads to an induced current whose direction is such that the induced magnetic field opposes the original flux change.

$$B_{Original} \rightarrow \frac{\Delta\Phi_B}{\Delta t} \rightarrow emf_{induced} \rightarrow I_{induced} \rightarrow B_{induced}$$

Lenz's Law: The direction of $B_{induced}$ is opposite the direction of $\Delta\Phi_B/\Delta t$ $\quad emf = -N\frac{\Delta\Phi_B}{\Delta t}$



B is into the board.

$\Delta\Phi_B/\Delta t$ is increasing → same direction as B → $\Delta\Phi_B/\Delta t$ is into the board.

B_I → opposite direction to $\Delta\Phi_B/\Delta t$ → B_I is out of the board.

Hence, I is CCW

Note: the RHR applied to a positive charge in the rod gives an upward force

Example: A circular coil (950 turns, 60.0 cm in radius) is rotating in a uniform magnetic field. At t = 0.00 s, the normal to the coil is perpendicular to the magnetic field. After the coil makes one eighth of a revolution in t = 10.0 ms, the normal to the coil makes an angle of 45° with the field. An average emf of 65.0 mV is induced in the coil during this time. What is the magnitude of the magnetic field?

$$|emf| = N\frac{\Delta\Phi_B}{\Delta t} = NAB\frac{Cos\theta_{final} - Cos\theta_{init}}{\Delta t}$$

$$B = \frac{|emf|\Delta t}{NA[Cos\theta_{final} - Cos\theta_{init}]} = \frac{(0.0650V)(0.0100s)}{(950)[\pi(0.600\text{m})^2][Cos45° - Cos90°]} = 85.6\mu\text{T}$$

Example: A piece of copper is formed into a single circular loop of radius 12.0 cm. A magnetic field is oriented parallel to the normal to the loop, and it increases from 0 to 0.600 T in a time of 0.450 s. The wire has a resistance per length of 3.30 x 10^{-2} Ω/m. What is the average electrical power dissipated by the resistance in the wire?

$$|emf| = N\frac{\Delta\Phi_B}{\Delta t} = NA\frac{B_{final} - B_{init}}{\Delta t}Cos\theta = (1)\pi(0.120m)^2\frac{0.600T\text{-}0T}{0.450s}(1) = 60.3186mV$$

$$R = L\left(\frac{R}{L}\right) = 2\pi(0.120m)(3.30\times10^{-2}\Omega\cdot m) = 0.02488\Omega$$

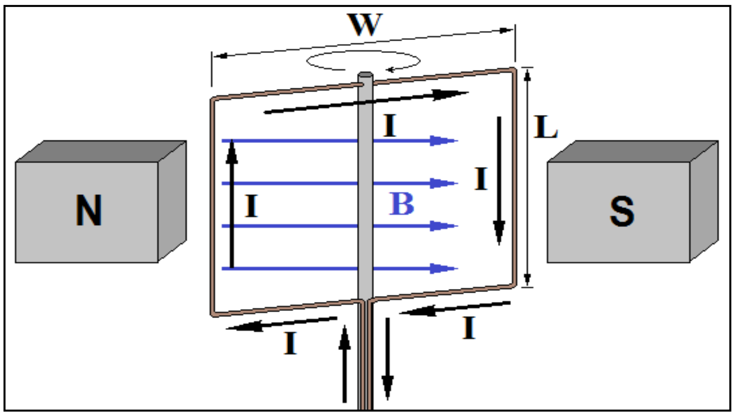
$$P = \frac{V^2}{R} = \frac{(0.0603186V)^2}{0.02488\Omega} = 0.146W$$

Example: A circular coil (950 turns, 60.0 cm in radius) is positioned in a magnetic field with the face of the coil perpendicular to a time-varying magnetic field, B = $\alpha t^2+\beta$, where α = 0.125 T/s^2 and β = 12.5 T. Determine the magnitude of the emf in the coil at t = 25.0 ms.

$$|emf| = N\frac{d\Phi_B}{dt} = NA\frac{dB}{dt}$$

$$|emf| = N(\pi r^2)(2\alpha t) = (950)\pi(0.600m)^2(2)(0.125T/s^2)(0.025s) = 6.72V$$

Electric Generator : $emf = NBA\omega Sin(\omega t)$



If we send current through the loop the magnetic field creates torque, turning the loop. This is a ***motor****.*

Instead of sending current through the loop, we rotate the loop in the field, generating an emf (and hence current). This is an ***electric generator****.*

$$emf = -N\frac{d\Phi_B}{dt} \qquad \Phi_B = BACos\theta$$

$$emf = -NAB\frac{d(Cos\theta)}{dt} = -NAB\frac{d(Cos\theta)}{d\theta}\cdot\frac{d\theta}{dt} = NABSin(\theta)\frac{d\theta}{dt} = NAB\omega Sin(\omega t)$$

Back EMF in Motors : $I = \frac{V - emf_{back}}{R}$

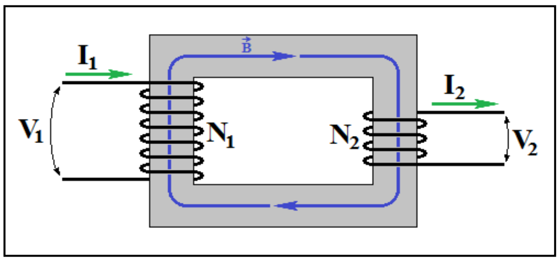
- Current through a motor's coil(s) experiences a force opposing the motion (**back emf**)
- Generator $\rightarrow$ coil rotates in B field generating emf and current
- Motor $\rightarrow$ emf and current in coil causes it to rotate in a B field.
- Once rotating, a motor acts as a generator, drawing power from the motor (Lenz's Law)

Example: When a motor powered by a 120V source first starts running it draws 39.3 A. Once it is running at full speed the motor draws 7.21 A. Determine the back emf at full speed.

$$R = \frac{V}{I} = \frac{120\text{V}}{39.3\text{A}} = 3.053435\Omega$$

$$emf_{back} = V - IR = (120V) - (7.21A)(3.053435\Omega) = 98V$$

Transformers: $\frac{V_1}{V_2} = \frac{N_1}{N_2}$ $\frac{I_1}{I_2} = \frac{N_2}{N_1}$



$V_1 = -N_1 \frac{\Delta\Phi_B}{\Delta t}$ $V_2 = -N_2 \frac{\Delta\Phi_B}{\Delta t}$

Now divide: $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

Turn Ratio → N_1:N_2

- Power Remains Constant: $I_1V_1 = I_2V_2$ $\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{N_1}{N_2}$
- For the Transformer to function it requires that F_B be changing → AC

Transmission Lines

- While the resistance of wires in circuits is negligible, in transmission lines where conductors may be miles long the losses are significant
- $P = I^2R$
- To minimize the losses, transformers are used to operate at high voltage (low current) across the transmission lines. Then stepped down at "load".