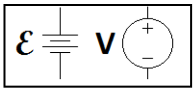
Part 4: Circuits

University Physics V2 (Openstax): Chapters 9 & 10
Physics for Engineers & Scientists (Giancoli): Chapters 25 & 26

Circuits:

- A **Circuit** is a closed path which allows electric current (electrons) to flow.
- Need a power source

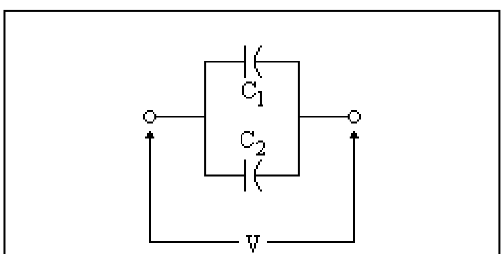


An electromotive force (emf or E) maintains a constant electric potential between its terminals (also called a "voltage source").

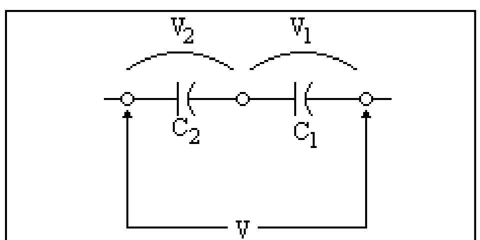
- Elements are connected via conductors (i.e. wires), which are represented in the circuit as lines.
- In conductors in static equilibrium, E=0.
    - This makes them equipotential.
    - There is no voltage drop/rise along a conductor.
- In conductors NOT in static equilibrium, usually E≈0.
    - The voltage drop/rise along them is negligible

Series and Parallel

- **Series Connection**: Elements are in the same path with no junction between them.
- The order of most elements (all we will see in this class) in series does not matter.
    - These may be rearranged to simplify a circuit.
- **Parallel Connection**: Elements have the same potential.
    - A direct path via conductors exists between the terminals of the two elements.
- A component or group of components may be replaced by another (known as an **equivalent**) if it has no effect on outside components.
    - The equivalent behaves (mathematically) the same as the original

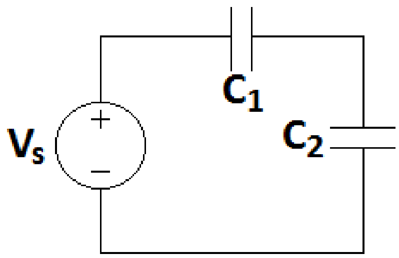


TWO CAPACITORS IN PARALLEL



TWO CAPACITORS IN SERIES

Capacitors in Series

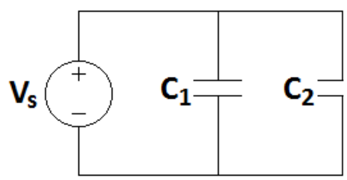


$$Q_1 = Q_2 = Q \quad V_s = V_1 + V_2$$

$$V_s = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)Q = \left(\frac{1}{C_{EQ}}\right)Q$$

$$\left(\frac{1}{C_{EQ}}\right) = \left(\frac{1}{C_1} + \frac{1}{C_2}\right) \qquad C_{EQ} = \frac{C_1 C_2}{C_1 + C_2}$$

Capacitors in Parallel

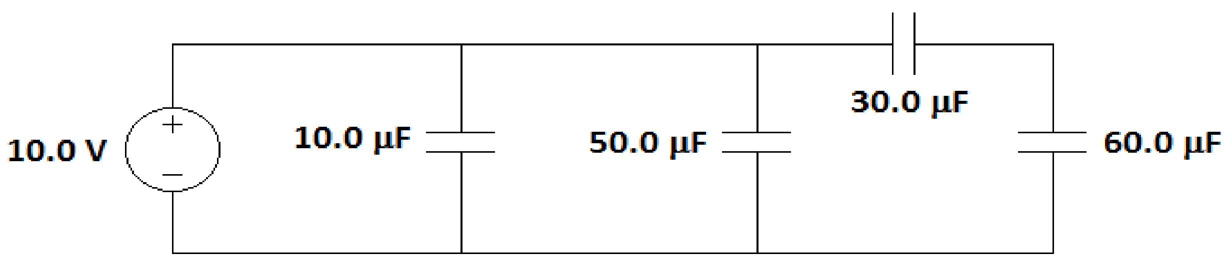


$$Q_1 + Q_2 = Q \quad V_s = V_1 = V_2$$

$$Q = Q_1 + Q_2 = C_1 V_s + C_2 V_s = (C_1 + C_2)V_s = C_{EQ}V$$

$$C_{EQ} = C_1 + C_2$$

Example: For the circuit shown, determine the equivalent capacitance and the charge on each capacitor.



The 30.0 mF and 60.0 mF capacitors are in series.

$$C_{EQ} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{30.0\times10^{-6}F} + \frac{1}{60.0\times10^{-6}F}\right)^{-1} = 20.0\mu F$$

The 10.0 mF, 50.0 mF and 20.0 mF capacitors are in parallel,

$$C_{EQ} = C_1 + C_2 + C_3 = 10.0\mu F + 50.0\mu F + 20.0\mu F = 80.0\mu F$$

$$Q = CV$$

$$Q_1 = (10.0\mu F)(10.0V) = 100\mu C$$

$$Q_2 = (50.0\mu F)(10.0V) = 500\mu C$$

Capacitors in series have the same charge.

$$Q_3 = Q_4 = (20.0\mu F)(10.0V) = 200\mu C$$

Current

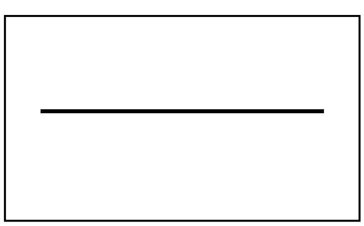
- An electric potential placed across a conductor, creates an electric field, which it turn causes charges to move. The movement of charges is called **electric current**. (not static equilibrium)
- $I = \frac{dq}{dt}$
- **"I"** is the electric current. "**dq**" is the amount of charge passing through a cross-sectional slice during the interval of time, "**dt**".
- Units of current: Ampere (Amp) 1 A = 1 C/s
- **Direct Current** (DC) – Current always flows the same direction.
- **Alternating Current** (AC) – Direction of current flow changes periodically
- Edison vs. Westinghouse – The "War of Currents"
- **"Conventional Current"** – Fictitious flow of positive charges.

Resistance

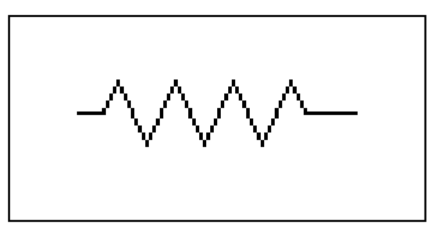
- Constant electric field ⇨ Constant acceleration in free space.
- But in materials electrons are slowed by interactions with atoms.
- Chances of a "collision" are proportional to the electron's speed.
- This behavior is similar to a velocity dependent frictional force.
- Using $\sum \vec{F} = m\vec{a}$, we get: $q\vec{E} - c\vec{v} = m\vec{a}$, where c is some constant.
- Electrons (on average) will accelerate until reaching "terminal velocity" (v=qE/c).
- Current is proportional to the electron velocity.
- The electron velocity is proportional to the electric field, E.
- The electric field, E, is proportional to the electric potential V.
- $I \propto v \propto E \propto V \rightarrow I \propto V$
- **Ohm's Law :** $V = IR$
  - V = Electric Potential/Voltage
  - I = Current
  - R= Resistance
- Units of Resistance: 1 Ohm (Ω) = 1V/A

Conductors/Resistors

- Resistance of conductors is very, very low.
- For example, 75m of 16 gauge copper wire → R≈1Ω
- A device with significant resistance is called a **resistor.**



Conductor Schematic

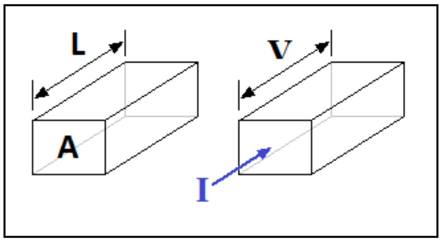


Resistor Schematic

Example: The filament of a light bulb has a resistance of 580Ω. A voltage of 120V is connected across the filament. How much current flows through the filament?

$$V = IR \qquad I = \frac{V}{R} = \frac{120V}{580\Omega} = 207mA$$

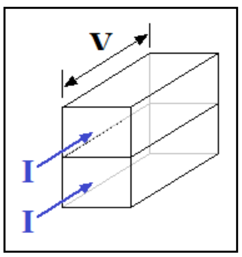
Resistance Changes With Length and Width



Start with a block of resistive material of Length (L) and cross-sectional area (A).

A voltage (V) applied across the block produces current (I).

The resistance of the block is: $R = \dfrac{V}{I}$

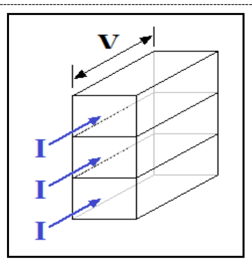


Place two blocks one on top of the other, doubling the area.

A voltage (V) applied across the block produces current (2I).

The resistance of the block: $R' = \dfrac{V}{2I} = \dfrac{1}{2}R$

Double the area and the resistance falls by a factor of 2.



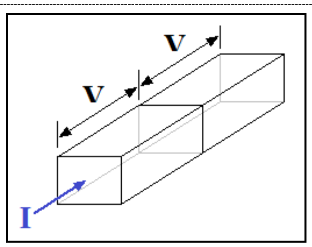
Place three blocks one on top of the other, tripling the area.

A voltage (V) applied across the block produces current (3I).

The resistance of the block: $R' = \dfrac{V}{3I} = \dfrac{1}{3}R$

Triple the area and the resistance falls by a factor of 3.

Resistance is inversely proportional to area. $R \propto \dfrac{1}{A}$

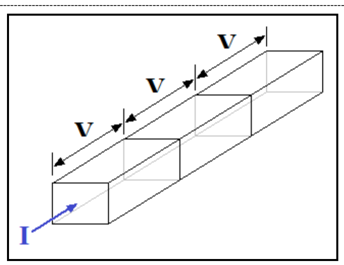


Place two blocks one after the other, doubling the length.

A current (I) through the blocks creates a voltage (2V)

The resistance of the block: $R' = \dfrac{2V}{I} = 2R$

Double the length and the resistance increases by a factor of 2.



Place three blocks one after the other, tripling the length.

A current (I) through the blocks creates a voltage (3V)

The resistance of the block: $R' = \dfrac{3V}{I} = 3R$

Triple the length and the resistance increases by a factor of 3.

Resistance is proportional to length. $R \propto L \qquad R \propto \dfrac{L}{A}$

Insert a constant and the proportionality becomes equality: $R = \rho \dfrac{L}{A}$

ρ = resistivity   Conductors $\rho \approx 10^{-8}\,\Omega\cdot$m   Insulators $\rho > 10^{10}\,\Omega\cdot$m

Example: A cylindrical copper cable carries a current of 1200A. There is a potential difference of 16 mV between two points on the cable that are 24 cm apart. What is the radius of the cable?

$$V = IR \qquad R = \frac{V}{I} = \frac{0.016V}{1200A} = 13.333\,\mu\Omega$$

$$R = \rho\frac{L}{A} = \rho\frac{L}{\pi r^2}$$

$$r = \sqrt{\frac{\rho L}{\pi R}} = \sqrt{\frac{(1.72\times10^{-8}\,\Omega\cdot m)(0.24m)}{\pi(13.333\times10^{-6}\,\Omega)}} = 9.9mm$$

Temperature and Resistivity : $\rho = \rho_0[1+\alpha(T-T_0)$ $R = R_0[1+\alpha(T-T_0)]$

- $\rho = \rho(T)$ = resistivity at temperature T
- $\rho_0 = \rho(T_0)$ = resistivity at temperature T_0
- α= "Temperature Coefficient of Resistivity"
- R= R(T) = resistance at temperature T
- $R_0 = R(T_0)$ = resistance at temperature T_0

NOTE: α is ONLY valid for a given value of T_0!

Example: The resistance of a platinum cylinder is 525Ω at 0.00°C. The temperature coefficient of resistivity for platinum is 0.00393 (°C)$^{-1}$ at 20.0 °C. What is the resistance of the cylinder at 35.0 °C?

$$R_0 = \frac{R}{1+\alpha(T-T_0)} = \frac{525\,\Omega}{1+0.00393(°C)^{-1}(0.00°C-20.0°C)} = 569.8\,\Omega$$

$$R = R_0[1+\alpha(T-T_0)] = 569.8\,\Omega[1+0.00393(°C)^{-1}(35.0°C-20.0°C)] = 603\,\Omega$$

Superconductors: When $T<T_C$, $\rho\to0$ T_C = "Critical Temperature".

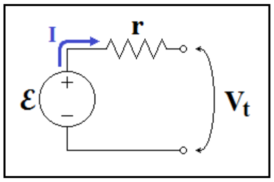
Electric Power: $P = IV$

- $P = \dfrac{W}{t} = \dfrac{qV}{t} = \dfrac{q}{t}V = IV$
- For Resistors: $P = IV = I^2R = \dfrac{V^2}{R}$
  - P = IV = I(IR) = I^2R
  - P = IV = (V/R)V = V^2/R

Example: When connected to an emf of 120 V, a light bulb consumes 60.0 W of power. Determine the resistance of the bulb and the current.

$$P = \frac{V^2}{R} \qquad R = \frac{V^2}{P} = \frac{(120V)^2}{60.0W} = 240\Omega \qquad\qquad P = IV \qquad I = \frac{P}{V} = \frac{60.0W}{120V} = 0.50A$$

Batteries

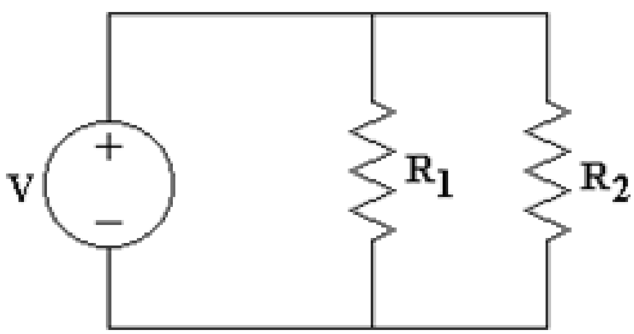


A battery is modeled by a series combination of an emf (voltage source) and a resistor (internal resistance).

Terminal Voltage (V_t): $V_t = E - Ir$

- Measure the voltage of your car battery:
  - When the car is off, the terminal voltage is $V_t \approx 12V$
  - When starting the car, the voltage drops: $V_t \approx 10V$
  - When the car is running, the voltage goes up: $V_t \approx 14V$
- When a battery goes bad, r typically becomes large.
- When a battery is drained, E drops.

Resistors in Parallel

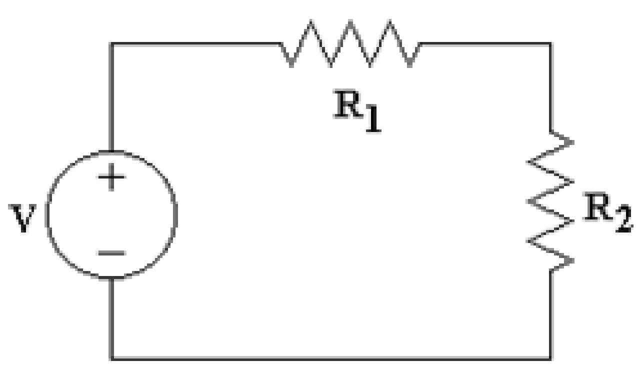


$$V_1 = V_2 = V_s \qquad I_s = I_1 + I_2$$

$$I_s = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)V = \left(\frac{1}{R_{EQ}}\right)V$$

$$\left(\frac{1}{R_{EQ}}\right) = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad R_{EQ} = \frac{R_1 R_2}{R_1 + R_2}$$

Resistors in Series



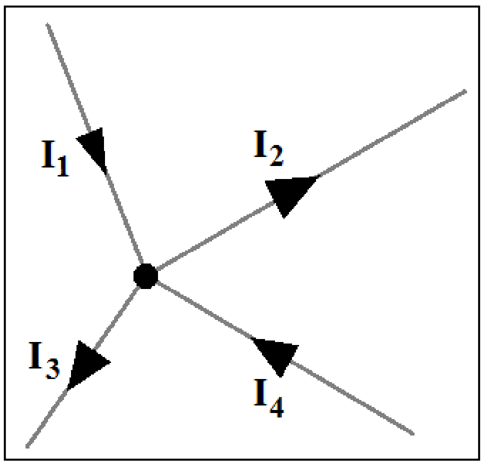
$$V_1 + V_2 = V_S \qquad I_s = I_1 = I_2$$

$$V_s = V_1 + V_2 = R_1 I + R_2 I = (R_1 + R_2)I = R_{EQ} I$$

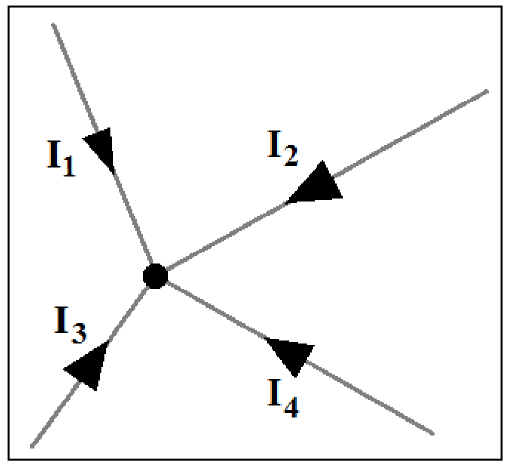
$$R_{EQ} = R_1 + R_2$$

Kirchoff's Junction Rule (Node Rule): The sum total of currents entering a junction (or node) is equal to the sum of the currents leaving that junction (or node).

- Junction - A point where two or more things are joined.
- Node - A point at which lines or pathways intersect or branch; a central or connecting point.
- This rule can be applied to any portion of a circuit that doesn't split circuit elements.
- As charge doesn't build up in a circuit (except locally in capacitors), this is simply conservation of charge.
- Current directions are often unknown. Just pick one. If wrong, that current will be negative when solved.



$I_1 + I_4 = I_2 + I_3$

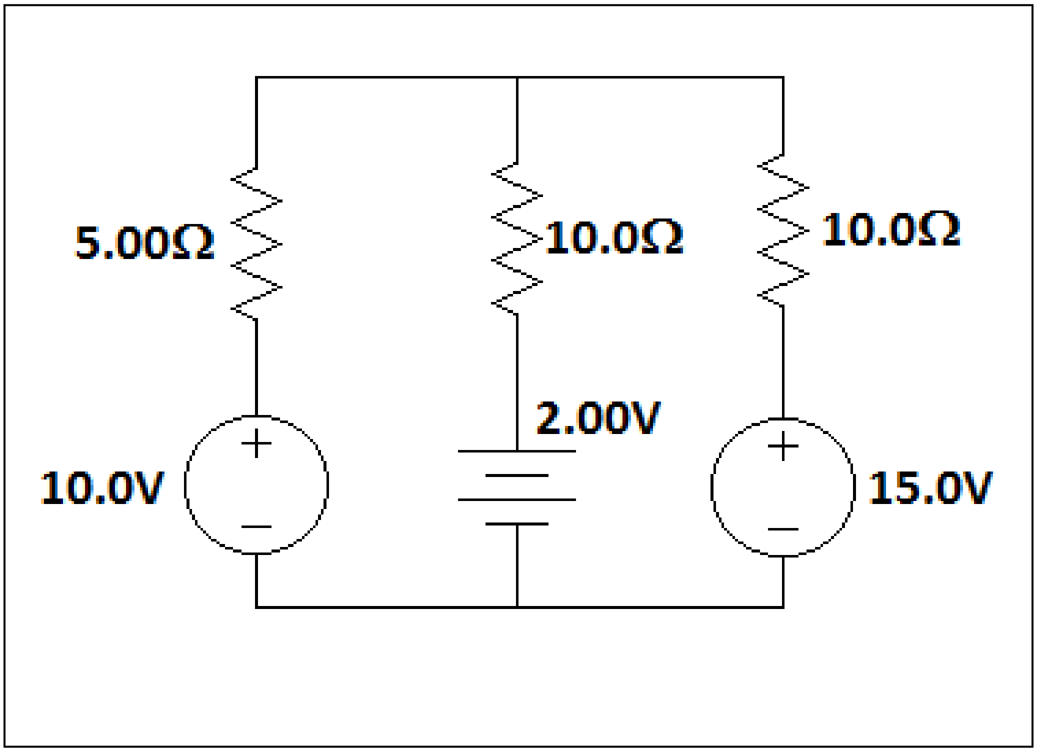


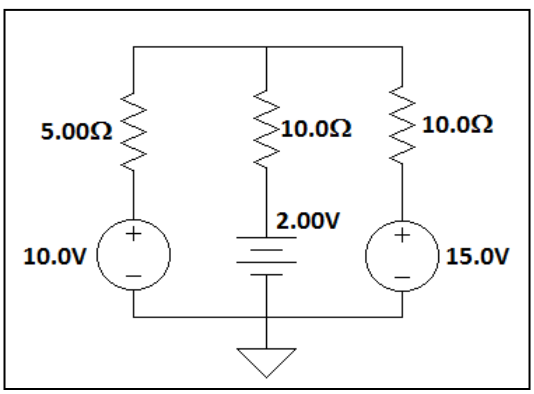
$I_1 + I_2 + I_3 + I_4 = 0$

Kirchoff's Loop Rule (Voltage): The sum total of the electric potential (voltages) around any closed loop is zero.

- This is essentially stating that an electron that returns to its original position must have the same potential energy (conserved).
- Current direction determines polarities of passive elements (for example resistors). Current enters at higher potential and leaves at lower potential. The sign on voltage must match choice of current direction.
- General Rules:
    - Assign a ground (a reference point where V=0), usually at the base of a voltage source.
    - Label node (junction) voltages.
    - Label currents in each branch
    - Mark resistor parity

Example: Determine the current through each branch of the circuit shown.

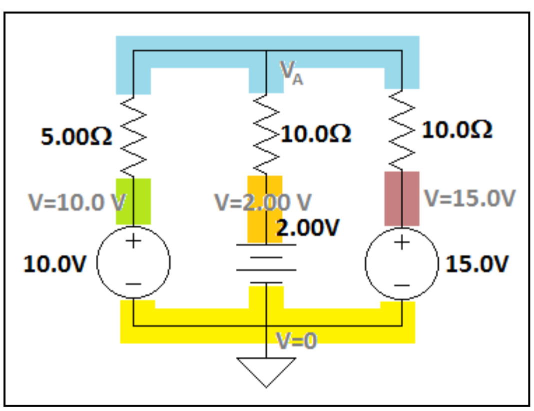




Step 1: Assign a ground

Typically the negative terminal of voltage sources are ideal candidates as ground.

The "common ground" is simply assigning a ground for calculation purposes, and has an inverted triangle as a schematic symbol.



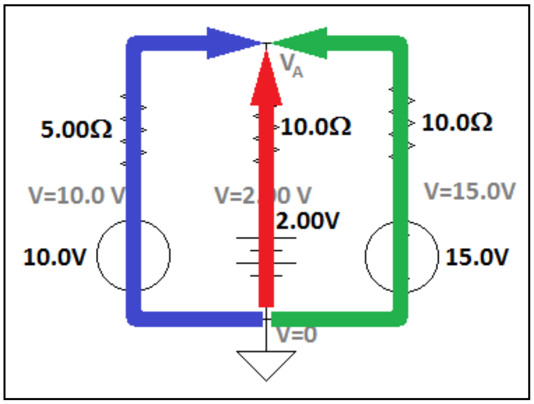
Step 2: Label node voltages

As there is a negligible voltage drop across the conductors, each colored region is equipotential.

The voltage of the "common ground" is set to zero.

The voltage at the positive terminal of a voltage source is the value of the source added to the voltage at the negative terminal of the source.

The unknown voltage is given a label (V_A).

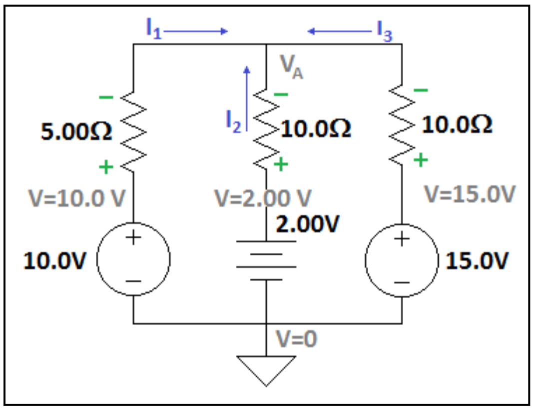


Step 3: Label current in each branch

Until solving a circuit, you can't be sure which direction the current will be flowing. So pick a direction (doesn't matter).

This circuit has three branches with three currents. I've drawn them all moving upwards, but you could make any of them move downward.

The current in the leftmost I am calling current I_1, then center path will carry the current I_2, and the rightmost path will carry current I_3.



Upon completion your circuit should look like this!

Step 4: Mark resistor parity

Place a "+" on the side of each resistor where the current enters. Place a "-" where the current leaves.

This is based on the direction in which you've drawn currents. Regardless of your choice of direction, the resistor polarity must match the current direction.

To get the voltage across any element, simply subtract the voltage on the "-" side from the voltage on the "+" side. The unknown voltage is given a label (V_A).

Now write equations and solve the circuit.

Kirchoff's Junction Rule: $I_1 + I_2 + I_3 = 0$

Kirchoff's Loop Rule: Follow any path that returns you to your starting point summing voltages along the way. Going from "-" to "+" is a "rise" (positive), and going from "+" to "-" is a "fall" (negative).

Up thru 10V source, down thru 2V source: $10.0V - (5.00\Omega)I_1 + (10.0\Omega)I_2 - 2.00V = 0$

Up thru 10V source, down thru 15V source: $10.0V - (5.00\Omega)I_1 + (10.0\Omega)I_3 - 15.0V = 0$

Up thru 2V source, down thru 15V source: $2.00V - (10.0\Omega)I_2 + (10.0\Omega)I_3 - 15.0V = 0$

Note: Only 2 of these are independent!

To get the voltage at any node in the circuit, start at a node with a known voltage and follow any path to the desired node, adding and subtracting the voltage of each element in the path.

$$V_A = 10.0V - (5.00\Omega)I_1 \qquad V_A = 15.0V - (10.0\Omega)I_3 \qquad V_A = 2.00V - (10.0\Omega)I_2$$

Solving the circuit: ① $I_1 + I_2 + I_3 = 0$

② $8.0V - (5.00\Omega)I_1 + (10.0\Omega)I_2 = 0$

③ $-5.0V - (5.00\Omega)I_1 + (10.0\Omega)I_3 = 0$

Solve ① for I_2 and substitute it into ②

$I_2 = -I_1 - I_3$ $\qquad 8.0V - (5.00\Omega)I_1 + (10.0\Omega)(-I_1 - I_3) = 0$

④ $8.0V - (15.00\Omega)I_1 - (10.0\Omega)I_3 = 0$

Solve ③ and ④ simultaneously to get I_1 or I_3. Adding equations ③ and ④ eliminates I_3.

$$3.0V - (20.00\Omega)I_1 = 0 \qquad I_1 = \frac{3.0V}{20.00\Omega} = 0.15\,\text{A}$$

Plug I_1 back into ② and ③ to get I_2 and I_3.

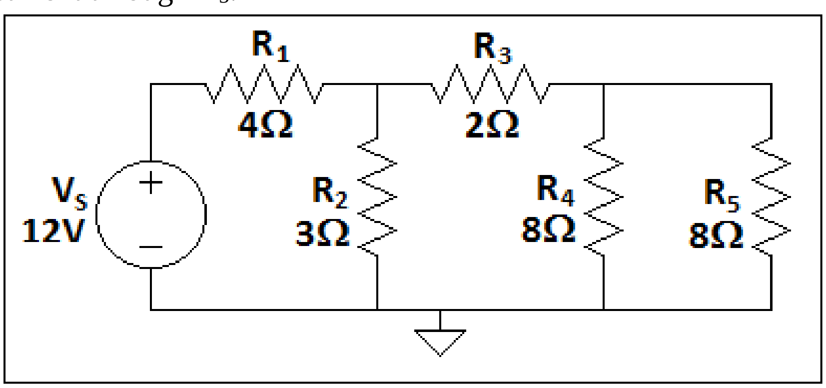
$$8.0V - (5.00\Omega)(0.15A) + (10.0\Omega)I_2 = 0 \qquad 7.25V + (10.0\Omega)I_2 = 0 \quad I_2 = -0.725A$$

$$-5.0V - (5.00\Omega)(0.15A) + (10.0\Omega)I_3 = 0 \qquad -5.75V + (10.0\Omega)I_3 = 0 \quad I_3 = 0.575A$$

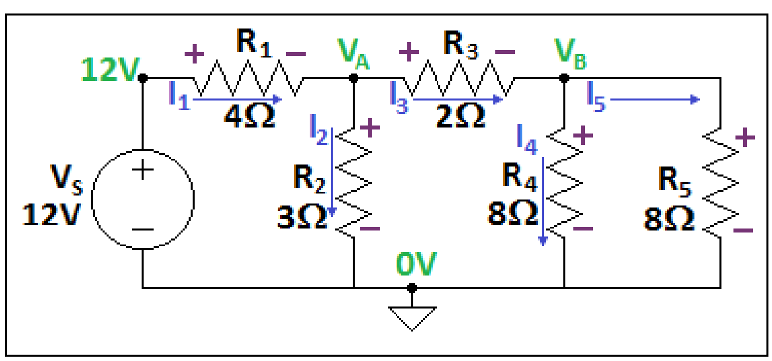
Check: $I_1 + I_2 + I_3 = 0.15A - 0.725A + 0.575A = 0$

We can also check the value of V_A: $V_A = 10.0V - (5.00\Omega)(0.15A) = 0.925V$

Example: Find the current through R_5.



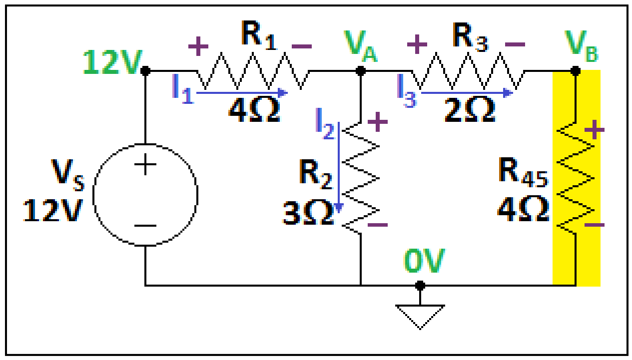
Assign a ground, label the node voltages, label the currents, and mark resistor parities.



This circuit we will solve by combining resistances. We start with R_4 and R_5.

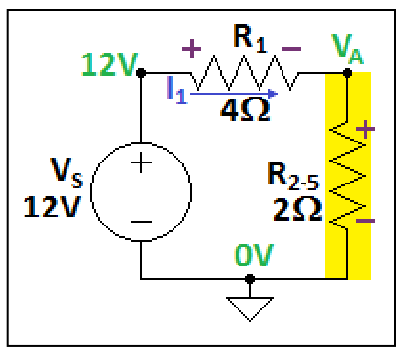
$$R_{45} = R_4 \parallel R_5 = \left(\frac{1}{8\Omega} + \frac{1}{8\Omega}\right)^{-1} = 4\Omega$$

The circuit now looks like this:



$$R_{345} = R_3 + R_{45} = 2\Omega + 4\Omega = 6\Omega \qquad R_{2-5} = R_2 \parallel R_{345} = \left(\frac{1}{6\Omega} + \frac{1}{3\Omega}\right)^{-1} = 2\Omega$$

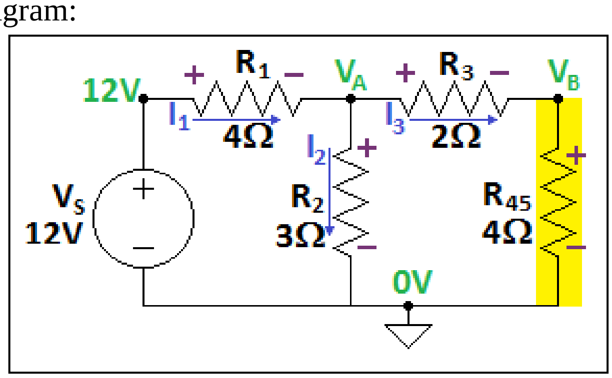
The circuit now looks like this:



$$R_{EQ} = R_1 + R_{2-5} = 4\Omega + 2\Omega = 6\Omega \qquad I_1 = \frac{V_S}{R_{EQ}} = \frac{12V}{6\Omega} = 2A$$

$$V_A = V_S - R_1 I_1 = 12V - (4\Omega)(2A) = 4V$$

Now back up one diagram:



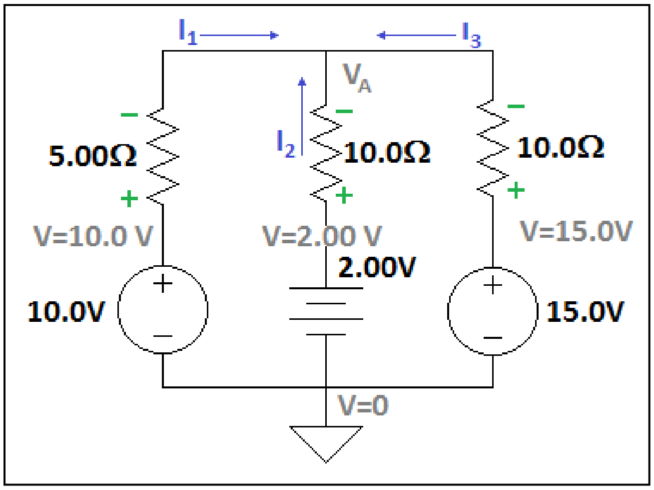
$$I_2 = \frac{V_A}{R_2} = \frac{4V}{3\Omega} = \frac{4}{3}A \qquad V_B = V_A - R_3 I_3 = 4V - (2\Omega)\left(\frac{2}{3}A\right) = \frac{8}{3}V$$

$$I_3 = I_1 - I_2 = 2A - \frac{4}{3}A = \frac{2}{3}A \qquad I_4 = I_5 = \frac{V_B}{R} = \frac{1}{R}V_B = \frac{1}{(8\Omega)}\left(\frac{8}{3}V\right) = \frac{1}{3}A$$

Nodal Analysis (Note: This is not in your textbook.)

- First, assign a ground, label the node voltages, label the currents, and mark resistor parities.
- Write an equation (or equations) using Kirchoff's junction rule.
- Use Ohm's law to write those currents in terms of new variables (node voltages)
- Solve for the node voltages and use them in Ohm's law to get currents.

We'll use a previous example as our circuit.



Kirchoff's Junction Rule: $I_1 + I_2 + I_3 = 0$

Write that equation in terms of node voltages: $I_R = \dfrac{V_+ - V_-}{R}$

$$\underbrace{\left(\frac{10.0V - V_A}{5.00\Omega}\right)}_{I_1} + \underbrace{\left(\frac{2.00V - V_A}{10.0\Omega}\right)}_{I_2} + \underbrace{\left(\frac{15.0V - V_A}{10.0\Omega}\right)}_{I_3} = 0$$

1 equation with 1 variable (V_A) ⇨ Solve for V_A.

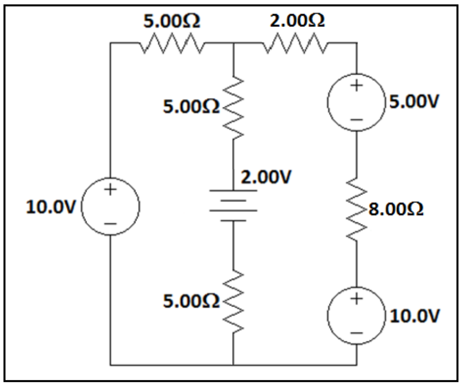
Multiply through by 10.0 Ω ⇨ $20 - 2V_A + 2 - V_A + 15 - V_A = 0$

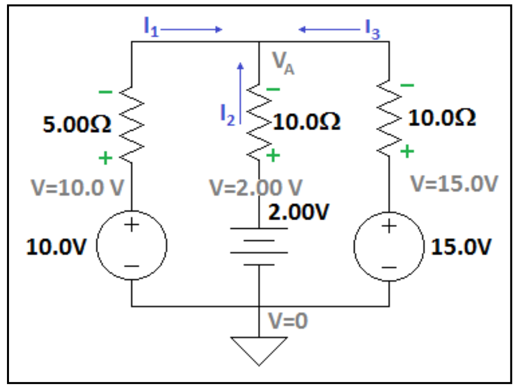
$37 - 4V_A = 0 \qquad V_A = 9.25V$

Find the Currents: $I_1 = \dfrac{10 - 9.25}{5} = 0.15A$

$$I_2 = \frac{2 - 9.25}{10} = -0.725A \qquad I_3 = \frac{15 - 9.25}{10} = 0.575A$$

How do we handle this? Combine resistors and voltage sources that are in series.

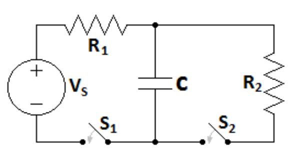




<u>Meters</u>

- <u>Ammeter</u>: measures current
  - Placed in series (so that current flows through it).
  - Very low resistance (so as not to affect the current it measures) .
  - **DO NOT** place in parallel! (You will blow a fuse)
- <u>Voltmeter</u>: measures voltage
  - Placed in parallel.
  - Very high resistance (so little current flows through it)
- <u>Multimeter</u>: can measure voltage or current (and often resistance)
  - Often a separate probe connection is used for ammeter (to prevent blowing fuses)
  - Typically the voltmeter is used preferentially (to prevent blowing fuses)

<u>RC Circuits</u>: Charging



Assume the capacitor C is uncharged when the switch S_1 closes at t=0. S_2 remains open.

Determine the capacitor current and voltage at some later time t.

- As S_2 is open, no current flows through it. The right branch can be disregarded. We have a single loop with V_S, R_1 and C.
- The charge on C builds gradually (charge doesn't just appear there). Thus, after the switch closes ($t=0^+$), V_C remains at zero (temporarily, until charge has a chance to build up on it).
  - With no voltage across C, the source voltage must appear across R (Kirchoff's loop rule).

$$V_S = V_R(t=0^+) = IR_1 \qquad I(t=0^+) = \frac{V_s}{R_1}$$

- If we wait a long time ($t \to \infty$), the circuit will reach equilibrium. In equilibrium, no current flows through the capacitor.

$$I(t \to \infty) = 0 \qquad V_C(t \to \infty) = V_S = \frac{Q(t \to \infty)}{C} \qquad Q(t \to \infty) = CV_S$$

- For $0 < t < \infty$, use Kirchoff's loop rule: $V_S - R_1 I - \dfrac{Q}{C} = 0$
- Letting I=dQ/dt results in a differential equation: $V_S - R_1 \dfrac{dQ}{dt} - \dfrac{Q}{C} = 0$
- The standard form for this is typically written as: $\dfrac{dQ}{dt} + \dfrac{Q}{R_1 C} = \dfrac{V_S}{R_1}$
- Next we need to separate the variables with Q on one side and t on the other.
  - $\dfrac{dQ}{dt} = \dfrac{V_S}{R_1} - \dfrac{Q}{R_1 C}$
  - $\dfrac{dQ}{dt} = \dfrac{-1}{R_1 C}\left(Q - CV_S\right)$
  - $\dfrac{dQ}{Q - CV_S} = \dfrac{-dt}{R_1 C}$
- Now integrate both sides and solve for Q.
  - $\displaystyle\int_0^Q \frac{dQ}{Q - CV_S} = \int_0^t \frac{-dt}{R_1 C}$
  - $\left[\ln(Q - CV_S)\right]_{Q=0}^{Q=Q} = \left[\dfrac{-t}{R_1 C}\right]_{t=0}^{t=t}$
  - $\ln\left(\dfrac{Q - CV_S}{-CV_S}\right) = \dfrac{-t}{R_1 C}$
  - $1 - \dfrac{Q}{CV_S} = e^{\frac{-t}{R_1 C}}$
  - $Q = CV_S\left\{1 - e^{\frac{-t}{R_1 C}}\right\}$
- This leads to the solution.

$$Q = CV_S\left\{1 - e^{\frac{-t}{R_1 C}}\right\} \qquad V_C = \frac{Q}{C} = V_S\left\{1 - e^{\frac{-t}{R_1 C}}\right\} \qquad I = \frac{V_S}{R_1} e^{\frac{-t}{R_1 C}}$$

- Time Constant: $\tau = R_1 C$

Example: A 12.0 V source is connected in series with a 36.0 μF capacitor, a 30.0 kΩ resistor, and an open switch. (a) How long after the switch is thrown will it take for the voltage across the capacitor to reach 6.00 V? (b) What is the voltage across the capacitor when the current is 100.0 μA?

$$V_C = V_{SS}\left\{1-e^{\frac{-t}{RC}}\right\} \qquad \frac{V_C}{V_{SS}} = 1-e^{\frac{-t}{RC}} \qquad e^{\frac{-t}{RC}} = 1-\frac{V_C}{V_{SS}}$$

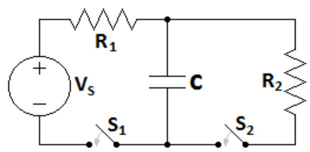
$$\frac{-t}{RC} = \ln\left(1-\frac{V_C}{V_{SS}}\right) \qquad \mathrm{t} = -RC\ln\left(1-\frac{V_C}{V_{SS}}\right)$$

$$\mathrm{t} = -(30.0\times10^{3}\,\Omega)(36.0\times10^{-6}\,F)\ln\left(1-\frac{6.00V}{12.0V}\right) = 0.749s$$

To get the current, we use Ohm's law.

$$V_C = V_S - IR = 12.0V - (100\times10^{-6}\,A)(30.0\times10^{3}\,\Omega) = 9.00V$$

RC Circuits: **Discharging**



With S_2 open, the capacitor C is charged to V_0 when the switch S_1 is opened at t<0. S_2 is the closed at some future time t=0.

Determine the capacitor current and voltage at some later time t.

- As S_1 is open, no current flows through it. The left branch can be disregarded. We have a single loop with R_2 and C.
- The charge on C builds gradually (charge doesn't just appear there). Thus, after the switch closes ($t=0^+$), V_C remains at V_0 (temporarily, until charge has a chance to leave it).
  - The capacitor voltage must also appear across R (Kirchoff's loop rule).

$$V_0 = V_R(t=0^+) = IR_2 \qquad I(t=0^+) = \frac{V_0}{R_2}$$

- If we wait a long time ($t \to \infty$), the circuit will reach equilibrium. In equilibrium, no current flows through the capacitor.

$$I(t\to\infty) = 0 \qquad V_C(t\to\infty) = V_R(t\to\infty) = 0 \qquad Q(t\to\infty) = 0$$

- For $0 < t < \infty$, use Kirchoff's loop rule: $\frac{Q}{C} - R_2 I = 0$
- Letting $I=-dQ/dt$ results in a differential equation: $\frac{Q}{C} + R_2\frac{dQ}{dt} = 0$
  - As a positive current reduces the charge on the capacitor, the minus sign is needed.
- The standard form for this is typically written as: $\frac{dQ}{dt} + \frac{Q}{R_2C} = 0$
- Next we need to separate the variables with Q on one side and t on the other.

- $\frac{dQ}{dt} = \frac{-Q}{R_2C}$
- $\frac{dQ}{Q} = \frac{-dt}{R_2C}$

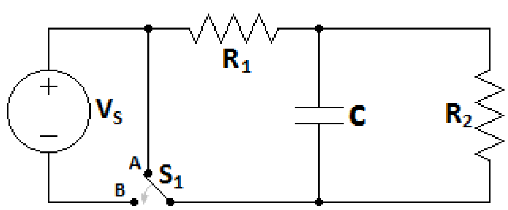
- Now integrate both sides and solve for Q.
  - $\int_{Q_0}^{Q} \frac{dQ}{Q} = \int_{0}^{t} \frac{-dt}{R_2C}$
  - $[\ln(Q)]_{Q=Q_0}^{Q=Q} = \left[\frac{-t}{R_2C}\right]_{t=0}^{t=t}$
  - $\ln\left(\frac{Q}{Q_0}\right) = \frac{-t}{R_2C}$
  - $\frac{Q}{Q_0} = e^{\frac{-t}{R_2C}}$

- This leads to the solution.

$$Q = Q_0 e^{\frac{-t}{R_2C}} = CV_0 e^{\frac{-t}{R_2C}} \qquad V_C = \frac{Q}{C} = V_0 e^{\frac{-t}{R_2C}} \qquad I = \frac{V_S}{R_2} e^{\frac{-t}{R_2C}}$$

- Time Constant: $\tau = R_2C$

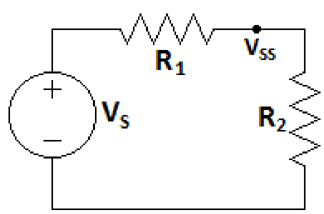
<u>Complex RC Circuits</u>: Charging



Assume the capacitor C is uncharged when the switch S_1 shifts from positions A to position B at t=0.

Determine the capacitor current and voltage at some later time t.

- In steady state, I_C=0 (open circuit)

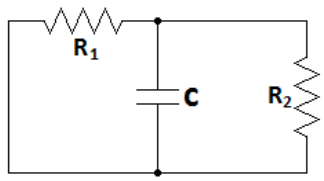


$$I = \frac{V_S}{R_1 + R_2}$$

$$V_{ss} = IR_2 = \frac{R_2}{R_1 + R_2} V_S$$

- For 0 < t < ∞, use: $V_C = V_{SS}\left\{1 - e^{\frac{-t}{RC}}\right\}$

- Use V_{SS} (the steady state voltage) and the equivalent resistance as seen from the capacitor.



To find R_{EQ}, short V_S and find R_{EQ} as seen from the capacitor.

$$R_{EQ} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

$$V_C = V_{SS}\left\{1 - e^{\frac{-t}{RC}}\right\} = \frac{R_2 V_S}{R_1 + R_2}\left\{1 - e^{\frac{-t(R_1+R_2)}{R_1 R_2 C}}\right\}$$

<u>AC Circuit Basics</u> $V = V_0 Sin(\omega t) = V_0 Sin(2\pi f t)$

- V_0 = "Peak voltage"
- ω = "Angular frequency"
- f = "Frequency"

- Example: Home (wall outlet): V_0 = 170 V f = 60 Hz
- Ohm's Law applies to functions and peak values.

$$I = \frac{V}{R} = \frac{V_0}{R} Sin(\omega t) = I_0 Sin(2\pi f t) \qquad V_0 = I_0 R$$

- Power: $P = IV = I_0 V_0 Sin^2(\omega t)$
  - As this oscillates, average power is preferred. $P_{AVG} = \frac{1}{2} I_0 V_0$
- Relying on RMS values ("root mean square") eliminates the half. $I_{RMS} = \frac{I_0}{\sqrt{2}}$ $V_{RMS} = \frac{V_0}{\sqrt{2}}$

$$P_{AVG} = \frac{1}{2} I_0 V_0 = \left(\frac{I_0}{\sqrt{2}}\right)\left(\frac{V_0}{\sqrt{2}}\right) = I_{RMS} V_{RMS}$$

- RMS values are also applicable to ohm's law and resistor power laws.

$$V_{RMS} = I_{RMS} R \qquad P = I_{RMS} V_{RMS} = \frac{V_{RMS}^2}{R} = I_{RMS}^2 R$$

- Example: Home (wall outlet): V_0 = 170 V $V_{RMS} = \frac{170V}{\sqrt{2}} = 120V$

<u>Physiological Effects of Current</u>

- ~1 mA results in "mild tingling"
- 10-20 mA results in muscle spasms
- ~200 mA is **potentially fatal** (Can cause heart fibrillations)
- Larger currents can stop the heart completely

<u>Grounding</u> : By connecting the outer casing of an appliance to ground, shocks are prevented by creating a path by which current from a loose internal wire will flow to ground.