Part 1: Electric Charges, Forces, and Fields

University Physics V1 (Openstax): Chapter 5
Physics for Engineers & Scientists (Giancoli): Chapter 21

Electric Forces and Charges

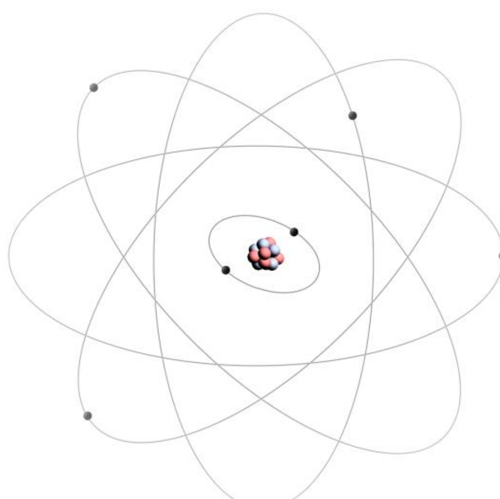
- Electric forces are created by electric charges.
- Electric charge is a property of subatomic particles.
- There are two types of charges (positive & negative).
- Like charges repel. Opposite charges attract.
- Electric charge is conserved and quantized.
- Unit of charge: Coulomb (C)
- "fundamental" charge: $e = 1.602\text{x}10^{-19}\text{C}$

Subatomic Particles

- Electron (-e)
- Proton (+e)
- Neutron (+0)

Atoms

- Proton (+e) and neutrons (+0) bound by the strong force create the positive nucleus.
- Negative electrons are bound to the nucleus via electrical attraction
- An atomic radius is $\sim 10^{-10}$m and a nuclear radius is $\sim 10^{-15}$m

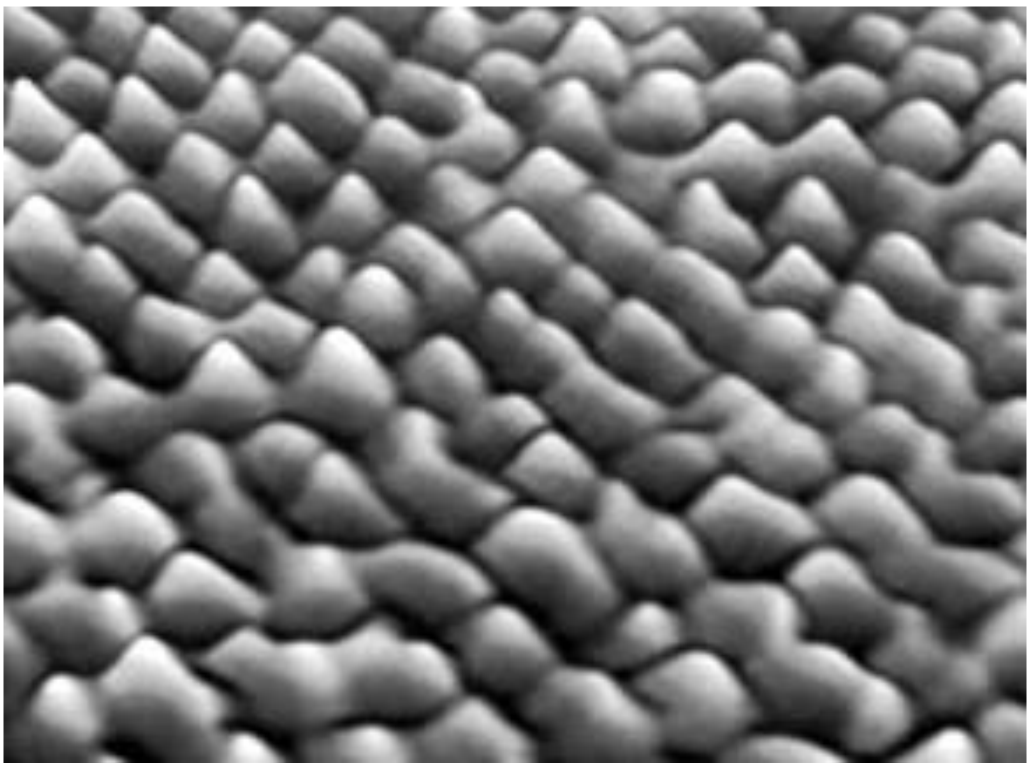


The carbon-12 atom

- Nucleus, charge +6
  - 6 protons (red spheres in the nucleus), charge +1
  - 6 neutrons (blue spheres in the nucleus), charge +0
- 6 electrons (black dots orbiting nucleus), charge -1 each
  - 2 inner (core) electrons (orbit closest to the nucleus)
  - 4 outer (valence) electrons (orbit furthest from the nucleus)
- Not to scale: If the nucleus were roughly one third of an inch in diameter (~ 8 mm), then the diameter of the valence electron orbits would be roughly one half of a mile (800 m).

Solids

- Positively charged nuclei and inner (core) electrons are bound tightly in place.
- The outer (valence) electrons aren't bound as tightly.
- A material's electrical properties are determined by the strength of the valence electron bonds
    - Conductors: Electric current flows freely (loosely bound valence electrons)
    - Insulators: Current does NOT flow readily (tightly bound valence electrons)
    - Semi-conductors:
        - Electric properties are generally determined by impurities
        - For "chips" we control the number of loosely bound valence electrons.
            - Some impurities create conducting regions
            - Other impurities create insulating regions
            - Intricate circuits are created by finely infusing different impurities



Scanning electron microscope image of atomic mica.
The region shown is approximately 6 nm by 6nm.

Static Electricity

- Rubbing two neutral objects together can displace electrons from one to the other.
- These charges will be equal in magnitude and opposite in sign.
- If given a conducting path, the charges will readily flow together and neutralize each other.
- On humid days, polar water molecules can strip electrons from the metal of cars (shocks more likely).

Example: How many electrons does it take to make -16μC?

Basically, it's a unit conversion: $$(-16\times10^{-6}C)\frac{1\text{ electron}}{(-1.6\times10^{-19}C)}=10^{14}\text{ electron}$$

Example: What is the net charge of an object that is made of $8.7\text{x}10^{28}$ protons, $1.05\text{x}10^{29}$ neutrons, and $8.699\text{x}10^{28}$ electrons?

Just add up all the charges:

$$Q_{NET} = (8.7\text{x}10^{28})(1.6\text{x}10^{-19}C) + (1.05\text{x}10^{29})(0) + (8.699\text{x}10^{28})(-1.6\text{x}10^{-19}C)$$

$$Q_{NET} = (8.7\text{x}10^{28} - 8.699\text{x}10^{28})(1.6\text{x}10^{-19}C) = 1.6MC$$

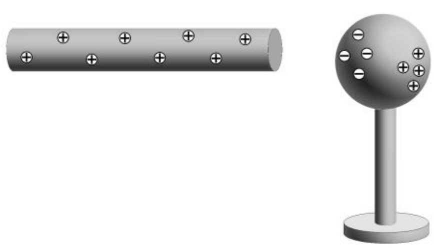
FYI: In this example we took 5 kg of Cu and removed one out of every 8700 electrons (slight imbalance).
The discharge of a lightning bolt is typically 15C to 350C.
1.6 MC ~ 10,000 lightning bolts

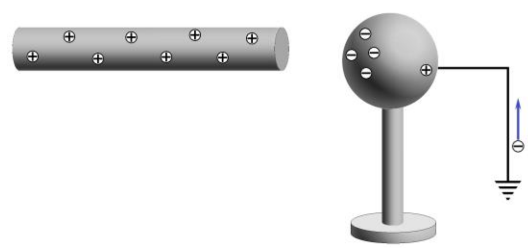
Electric Forces vs. Gravity

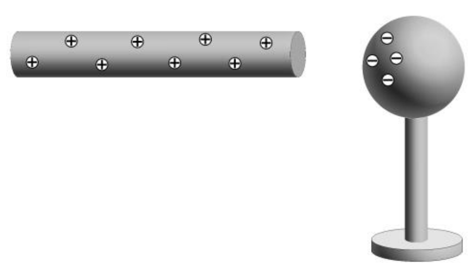
- Electromagnetic forces are much stronger than gravity.
- Apart from gravity, every force we studied in physics I was electromagnetic in nature.
  - Contact is an illusion (repulsion of outer electrons)
- The repulsion of relatively few electrons at the bottom of my shoes is supporting me when the mass of the entire Earth is pulling me down.

Inducing Charge

- A charged object (rod) is moved near to a conductor (sphere). This polarizes the free charges in the conductor (opposite charges are attracted while like charges are repelled). -Left Image-
- A grounded conducting wire is then touched to the opposite side of the sphere, allowing electrons to enter and neutralize the positive charges. -Right Image-
- When the conducting wire is removed, we are left with charge on the sphere. -Bottom Image-



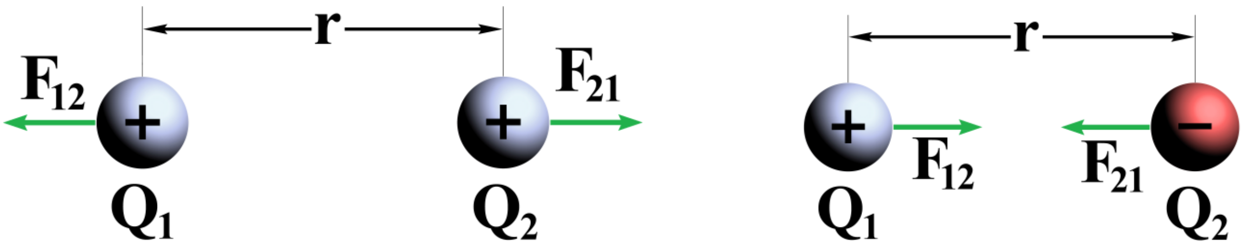




Coulomb’s Law: $F = k\frac{|Q_1||Q_2|}{r^2}$

- **F** is the magnitude of the force felt by both charges
- **k** is Coulomb’s constant ($k = 9.0\text{x}10^9\ Nm^2/C^2$)
- $\mathbf{Q_1}$ and $\mathbf{Q_2}$ are the two charges
- **r** is the separation between the charges

- Coulomb's law only finds the magnitude of the force ⇨ only use the magnitude of the charges.
- Typically the absolute value signs on the charges are left out (magnitude is assumed).
- The forces are directed along the line connecting the point charges.



Notation: F_{12} is the force on Q_1 due to Q_2

Coulomb vs. Newton : $F = k\dfrac{|Q_1||Q_2|}{r^2}$ vs. $F = G\dfrac{m_1 m_2}{r^2}$

- Both are "inverse square laws" (very similar).
- Masses in Newton's Law play the role of charges in Coulomb's Law.
- Constants: $k = 9.0\text{x}10^9 \text{Nm}^2/\text{C}^2$ $G = 6.67\text{x}10^{-11} \text{Nm}^2/\text{kg}^2$
- Let's compare the forces on two protons separated by a distance of 1.00 m
  - Gravity: $F = (6.67\text{x}10^{-11}\text{Nm}^2/\text{kg}^2)(1.67\text{x}10^{-27}\text{kg})^2/(1.00\text{m})^2 = 1.86\text{x}10^{-64}\text{N}$
  - Electric: $F = (9.0\text{x}10^{9}\text{Nm}^2/\text{C}^2)(1.6\text{x}10^{-19}\text{C})^2/(1.00\text{m})^2 = 2.3\text{x}10^{-28}\text{N}$
  - The electric force is 36 orders of magnitude stronger!
- Large Scale: Electric charges neutralize each other, gravity dominates.

Example: Two tiny conducting spheres are identical, but carry differing charges of -20.0 μC and 50.0 μC. They are separated by a distance of 2.50 cm. (a) What is the magnitude of the force that each sphere experiences and is it attractive or repulsive? And (b) if the spheres are briefly brought into contact and then returned to a separation of 2.50 cm, what is the magnitude of the force that each sphere experiences and is it attractive or repulsive?

(a) $$F = k\frac{|Q_1||Q_2|}{r^2}$$

$$F = \left(9.0\times10^9\ \frac{\text{Nm}^2}{\text{C}^2}\right)\frac{(20.0\times10^{-6}\,\text{C})(50.0\times10^{-6}\,\text{C})}{(0.0250\,\text{m})^2} = 14.4kN \quad \text{(Attractive)}$$

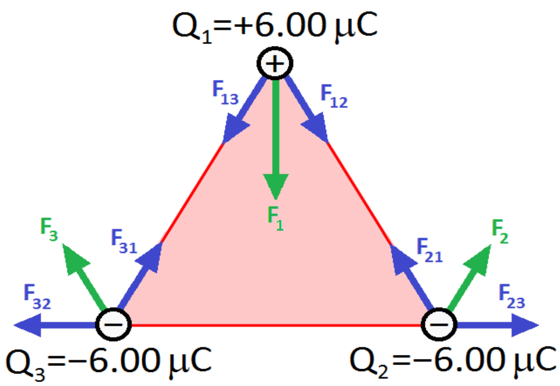
(b) Contact allows charges to mix (annihilate).
Half of the remaining charge distributes on each sphere.

$Q_1 = Q_2 = ½(-20.0\ \mu\text{C} + 50.0\ \mu\text{C}) = 15.0\ \mu\text{C}$

$$F = k\frac{|Q_1||Q_2|}{r^2}$$

$$F = \left(9.0\times10^9\ \frac{\text{Nm}^2}{\text{C}^2}\right)\frac{(15.0\times10^{-6}\,\text{C})^2}{(0.0250\,\text{m})^2} = 3.24kN \quad \text{(Repulsive)}$$

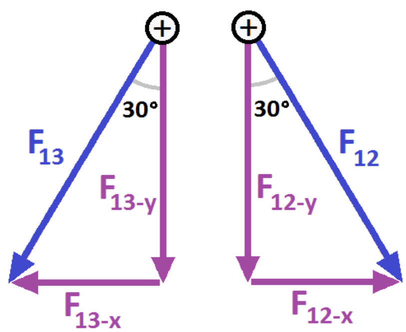
Example: Determine the magnitude and direction of the net force on each of the three charges due to the other two. Each charge is at the corner of an equilateral triangle with sides of length 1.20 m.



As the magnitude of the charges is the same, and they are separated by the same distance, then the magnitude of the forces will all be the same.

$$F_{12} = F_{13} = F_{21} = F_{23} = F_{31} = F_{32} = F = k\frac{Q^2}{r^2}$$

$$F = \left(9.0\times10^{9}\,\frac{Nm^2}{C^2}\right)\frac{(6.00\times10^{-6}\,C)^2}{(1.20m)^2} = 0.225\,N$$

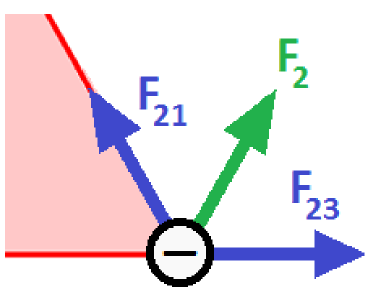


Due to symmetry we know F_1 is vertical.

$$F_1 = F_{13\text{-}y} + F_{12\text{-}y} = 2F_{13\text{-}y} = 2F\cos 30° =$$

$$= 2(0.225N)\cos 30° = 0.39\,N$$

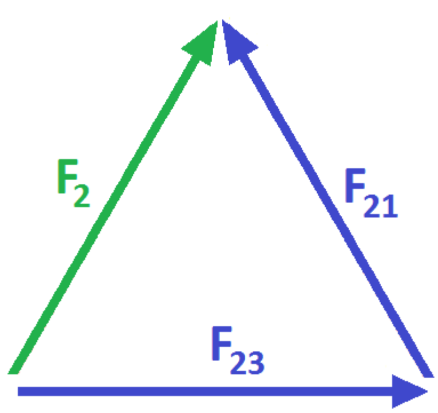
F_1 = 0.39N at 270°



Symmetry can also help with F_2 and F_3

As F_{23} and F_{21} have the same magnitude, F_2 must be directed halfway in between F_{23} and F_{21}.

$\theta_2 = ½(180^o-60^o) = 60^o$



By drawing the vector addition, you will then discover that you have an equilateral triangle.

All the sides are the same. ⇨ F_2 = 0.225N at 60°

By symmetry ("flipping" around y-axis) we can determine that F_3 must have the same magnitude as F_2, and the direction is easy to determine.

F_3 = 0.225N at 120°

Fields

- Instead of acting directly on each other, objects create fields, and these fields act upon other objects to create forces.
- You are already familiar with the gravitational field.
- To calculate the force of gravity for objects on the surface of the Earth we could use Newton's law.

$$F = G\frac{M_E m}{R_E^2} = m\left[G\frac{M_E}{R_E^2}\right] = mg$$

- "Gravitational Field": $g = G\dfrac{M_E}{R_E^2}$
- These are "vector fields", as they consist of a different vector (magnitude and direction) at every point in space.

In other words, a vector field is a collection of an infinite number of vectors,

with a distinct vector at each point in space.

Electric Fields : $\vec{E} = \dfrac{\vec{F}}{q}$

- **q** is a small positive test charge (at some point in the field)
- **F** is the force felt by that charges
- **E** is the electric field at that point.

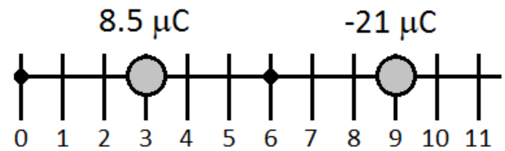
- **E** and **F** are vectors. **E** points in the same direction as the force felt by a positive test charge.
- The unit of the electric field is $\dfrac{N}{C}$ (Note: $1\ \dfrac{N}{C} = 1\ \dfrac{V}{m}$ "volt per meter")
- For a point charge Q : $\vec{E} = \dfrac{1}{q}\vec{F} = \dfrac{1}{q}\left(\dfrac{kQq}{r^2}\right) = \dfrac{kQ}{r^2}$
- Like forces, electric fields obey **superposition** (add the vectors up)

Example: An electric field of strength 260 kN/C points due west at a certain spot. What are the magnitude and direction of the force that acts on a -7.0 μC charge at this spot?

$F = qE = (7.0\times10^{-6}\,C)(260{,}000\ N/C) = 1.8\ N$
The field points to the west.
The force on a positive charge is to the west.
The force on a negative charge is to the east.

F = 1.8 N (pointing east)

Example: Two charges are placed on the x-axis. One charge (Q_1 = +8.5 μC) is at x_1 = +3.0 cm, and the other (Q_2 = -21 μC) is at x_2 = +9.0 cm. Find the net electric field (magnitude and direction) at (a) x = 0 cm, and (b) x = +6.0 cm.



$$\vec{E} = \frac{kQ}{r^2}$$ (…but sign must be determined by determining field direction)

$$\vec{E}_1(x=0) = -\left(9.0\times10^9\ \frac{\text{Nm}^2}{\text{C}^2}\right)\frac{(8.5\times10^{-6}\,\text{C})}{(0.030\text{m})^2} = -8.5\times10^7\ \frac{\text{N}}{\text{C}}$$

$$\vec{E}_1(x=6) = \left(9.0\times10^9\ \frac{\text{Nm}^2}{\text{C}^2}\right)\frac{(8.5\times10^{-6}\,\text{C})}{(0.030\text{m})^2} = 8.5\times10^7\ \frac{\text{N}}{\text{C}}$$

$$\vec{E}_2(x=0) = \left(9.0\times10^9\ \frac{\text{Nm}^2}{\text{C}^2}\right)\frac{(21\times10^{-6}\,\text{C})}{(0.090\text{m})^2} = 2.3\times10^7\ \frac{\text{N}}{\text{C}}$$

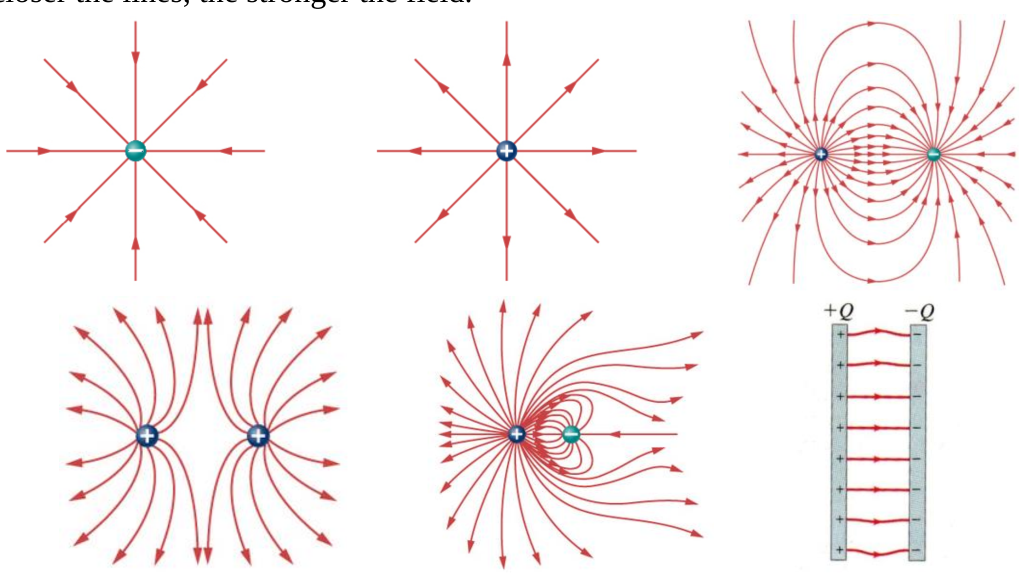
$$\vec{E}_2(x=6) = \left(9.0\times10^9\ \frac{\text{Nm}^2}{\text{C}^2}\right)\frac{(21\times10^{-6}\,\text{C})}{(0.030\text{m})^2} = 21\times10^7\ \frac{\text{N}}{\text{C}}$$

(a) $\vec{E}(x=0) = E_1 + E_2 = -8.5\times10^7\ \frac{\text{N}}{\text{C}} + 2.3\times10^7\ \frac{\text{N}}{\text{C}} = -6.2\times10^7\ \frac{\text{N}}{\text{C}}$

(b) $\vec{E}(x=6) = E_1 + E_2 = 8.5\times10^7\ \frac{\text{N}}{\text{C}} + 21\times10^7\ \frac{\text{N}}{\text{C}} = 3.0\times10^8\ \frac{\text{N}}{\text{C}}$

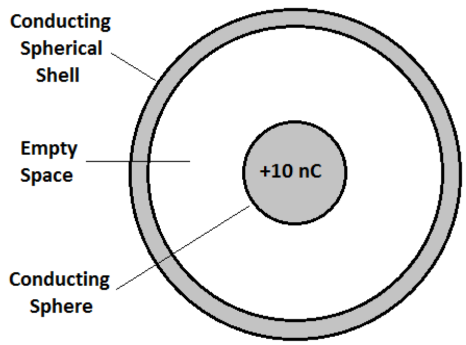
Electric Field Lines

- Lines indicate the direction the field vectors point (field vectors are tangent to the field lines).
- The number of lines starting on a positive charge (or ending on a negative charge) is proportional to the charge.
- The closer the lines, the stronger the field.



Electric Fields And Conductors In Static Equilibrium

- Inside a conductor, E=0.
- Any net charge distributes itself evenly on the surface.
- At the surface of the conductor, the electric field is perpendicular to that surface.



Inside sphere: no net charge, E=0.

Sphere surface: 10nC evenly distributed.

Shell inner surface: -10nC evenly distributed

Inside shell: no net charge, E=0.

Shell outer surface: 10nC evenly distributed.

Empty space: Same electric field as if a 10nC point charge was at the center of the sphere.

Motion of a Charged Particle in an Electric Field: $\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$

- If you are given the field, use that to find the acceleration and then solve the kinematics.
- Given kinematic information, find the acceleration, then use a=qE/m to find q or E.

Example: An electron is accelerated in the uniform field (E = $1.45\text{x}10^4$ N/C) between two parallel charged plates. The separation between the plates is 1.10 cm. The electron is accelerated from rest near the negative plate through a tiny hole in the positive plate. With what speed does it leave the hole?

$$\vec{a} = \frac{q\vec{E}}{m} = \frac{(1.6\times10^{-19}C)\left(1.45\times10^{4}\frac{N}{C}\right)}{9.11\times10^{-31}kg} = 2.54665\times10^{15}\frac{m}{s^2}$$

$x_0 = 0$ $\quad$ x = 0.0110 m $\quad$ $v_0 = 0$ $\quad$ v = ? $\quad$ a = $2.54665\text{x}10^{15}$ m/s^2

$$v^2 = v_0^2 + 2a(x - x_0) = 2ax$$

$$v = \sqrt{2ax} = \sqrt{2(2.54665\times10^{15})(0.0110m)} = 7.49\times10^{6}\,m/s$$

Continuous Charge Distributions

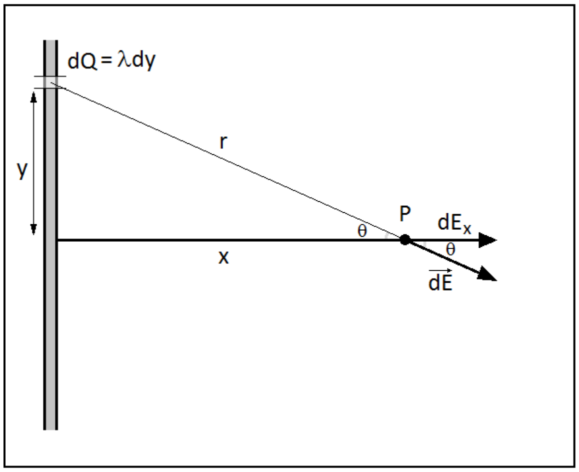
- Break the distribution into infinitesimally small point charges, determine the electric field from each, and add them up.

$$dE = k\frac{dQ}{r^2} \qquad \vec{E} = \int d\vec{E}$$

- NOTE: When integrating over vectors, you must do this as you would add vectors (by components).

$$E_x = \int dE_x \qquad E_y = \int dE_y \qquad E = \sqrt{E_y^2 + E_x^2}$$

Example: Determine the magnitude of the electric field at a point P a distance x from the midpoint of a very long line of uniformly distributed positive charge. Assume x is much smaller than the length of the wire. Let λ be the charge per unit length (units C/m).



First, select an infinitesimal part of the charge distribution. Label it "dQ".

Write "dQ" in terms of your integration variable. In this case dQ = λdy

Find dE. $dE = k\frac{dQ}{r^2} = k\frac{\lambda dy}{r^2}$

Can we slap this in an integral and get E?

NO! dE is a vector! $E = \int dE_x = \int dECos\theta$

$$E = \int dE_x = \int dECos\theta = \int\left(k\frac{\lambda dy}{r^2}\right)\left(\frac{x}{r}\right) = k\lambda x\int\frac{dy}{r^3} = k\lambda x\int\frac{dy}{\left[x^2+y^2\right]^{3/2}}$$

Limits of integration? For generality, -L/2 to L/2

$$E = k\lambda x\int_{-L/2}^{L/2}\frac{dy}{\left[x^2+y^2\right]^{3/2}} = 2k\lambda x\int_{0}^{L/2}\frac{dy}{\left[x^2+y^2\right]^{3/2}}$$

Look it up on an integral table:

$$\int(y^2+a^2)^{-3/2}dy = \frac{y}{a^2\sqrt{y^2+a^2}}$$

$$E = 2k\lambda x\int_{0}^{L/2}\frac{dy}{\left[x^2+y^2\right]^{3/2}} = 2k\lambda x\left[\frac{y}{x^2\sqrt{y^2+x^2}}\right]_{y=0}^{y=L/2} = \frac{k\lambda L}{x\sqrt{(L/2)^2+x^2}}$$

L>>x, so let L go to infinity: $E = \frac{2k\lambda}{x}$ (answer)

What if λ = λ (y)?

$$E = kx\int\frac{\lambda(y)dy}{r^3} = kx\int\frac{\lambda(y)dy}{\left[x^2+y^2\right]^{3/2}}$$

What if my charge is distributed over an area or volume?

$dQ = \sigma dA$ $\quad dA = dxdy$ or $dA = dr\cdot rd\theta = rdrd\theta$

$dQ = \rho dV$

$dV = dxdydz$ or $dV = rdrd\theta dZ$ or $dV = r^2\sin\theta drd\theta d\phi$

Exercises

1. When a child rubs a balloon on his head, 5.00×10^{10} electrons are transferred from the child's hair to the balloon. Assuming both start electrically neutral, determine the net charge on (A) the balloon, and (B) the child's hair.

2. A 50.0 μC charge and a 40.00 μC Charge are separated by 3.00 m. (A) Determine magnitude of the net force each charge feels due to the presence of the other charge. (B) Are these forces attractive or repulsive?

3. Two 25.0 μC charges are fixed on the x-axis. One is placed at the origin, and the other is placed at x_2 = 6.00 m. A third object with a 50.0 mg mass and a charge of 40.0 μC is placed on the x-axis at x_3 = 3.00 m. Determine the acceleration of the third charge due to the electric forces between it and the first two charges.

4. A charge (Q_1 = 25.0 μC) is place at the point (1.00 m, -2.00 m) on an x-y plane. A second charge (Q_2 = 100 μC) is placed at the point (4.00 m, 2.00m). Assume these are the only two charges in the area. (A) Determine the direction of the electric force acting on charge Q_2. (B) Determine the magnitude of the electric force acting on charge Q_2. (C) Determine the x-component of the electric force acting on charge Q_2. (D) Determine the y-component of the electric force acting on charge Q_2. (E) Determine the direction of the electric force acting on charge Q_1. (F) Determine the magnitude of the electric force acting on charge Q_1. (G) Determine the x-component of the electric force acting on charge Q_1. (H) Determine the y-component of the electric force acting on charge Q_1.

5. Three charges are placed on an xy-plane. A 5.00 μC charge (Q_1) is fixed at the origin. A second charge (Q_2 = -10.0 μC) is fixed at the point (7.00 m, 1.00 m). A third charge (Q_3 = 5.00 μC) is placed at the point (3.00 m, 4.00 m). Determine (A) the x-component, and (B) the y-component of the net force acting on charge Q_3.

6. Three identical conducting spheres are sitting in a lab, each on stand that is a perfect electric insulator. The first sphere has a net charge of Q_1 = 55.0 μC. The second sphere has a net charge of Q_2 = 33.0 μC, and the third sphere has a net charge of Q_3 = 20.0 μC. The first two spheres are briefly held together (touching) and then separated. Then the second and third sphere are briefly held together (touching) and then separated. After this is done, determine the electric repulsion between the first and the third charge if they are placed 4.00 m apart.

7. When a small object is placed in an electric field with magnitude 3.00 kN/C, it experiences a force of 51.0 nN. What is the charge on that object?

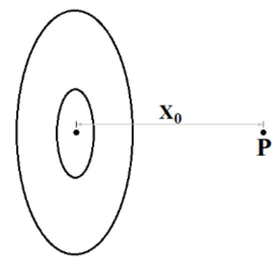
8. How far from a 25.0 nC charge is the magnitude of its electric field 9.00 N/C?

9. A charge (Q_1 = 200 nC) is placed at the origin. A second charge (Q_2 = -300 nC) is placed on the y-axis at y = 12.0 m. Assuming there are no other charges nearby, determine the magnitude of the net electric field at the point P (8.00 m, 6.00 m).

10. Determine (A) the magnitude of the acceleration, and (B) the direction of the acceleration of an electron that is placed in an electric field given by $\vec{E} = \left(4.00\frac{\mu N}{C}\right)\hat{\imath} + \left(3.00\frac{\mu N}{C}\right)\hat{\jmath}$.

11. One end of a thin rod holding charge Q is placed at the origin, and it extends along the x-axis to x_1 = 5.00 cm. Set up the integrals to find (A) the x-component of the electric field and (B) the y-component of the electric field at the point P = (x_2, y_2) = (8.00 cm, 3.00 cm).

12. A non-conducting annulus (the area between two concentric circles) with inner radius R_1 and outer radius R_2, has a charge Q uniformly distributed on one face. Determine the magnitude of the electric field at a point P, which is distance x_0 away from the center of the annulus along its rotational axis.



Exercise Solutions

1. When a child rubs a balloon on his head, 5.00×10^{10} electrons are transferred from the child's hair to the balloon. Assuming both start electrically neutral, determine the net charge on (A) the balloon, and (B) the child's hair.

   A. $(5.00 \times 10^{10}\ electrons)\left(\frac{-1.60 \times 10^{-19}\ C}{electron}\right) = -8.00\ nC$

   B. 8.00 nC *Charge is conserved. The net charge before is zero. It must be the same after.*

2. A 50.0 μC charge and a 40.00 μC Charge are separated by 3.00 m. (A) Determine magnitude of the net force each charge feels due to the presence of the other charge. (B) Are these forces attractive or repulsive?

   A. $F = k\frac{Q_1 Q_2}{r^2} = \left(9.00 \times 10^9\ \frac{N\cdot m^2}{C^2}\right)\frac{(50.0 \times 10^{-6}\ C)(40.0 \times 10^{-6}\ C)}{(3.00\ m)^2} = 2.00\ N$

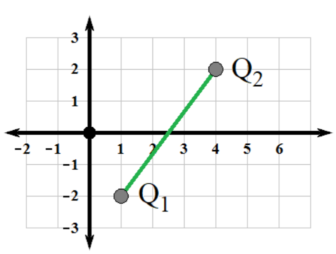
   B. Repulsive *Both charges are positive. Charges with the same sign repel.*

3. Two 25.0 μC charges are fixed on the x-axis. One is placed at the origin, and the other is placed at $x_2 = 6.00$ m. A third object with a 50.0 mg mass and a charge of 40.0 μC is placed on the x-axis at $x_3 = 3.00$ m. Determine the acceleration of the third charge due to the electric forces between it and the first two charges.

   The answer is zero. *This can be shown with symmetry. Rotating the problem (mirror image) around the point x = 3 cm doesn't change the problem (because the first two charges are identical). This means that the answer to this problem must also remain the same after this same rotation. Electric forces are vectors that line on the line connecting the two points, limiting it to either being in the positive x-direction or the negative x-direction. However, if it has any magnitude other than zero, it will change direction after the rotation. Consequently, it must be zero. If the net force is zero, then so is the acceleration. This logic only holds true for vectors. In the future we will calculate scalars. These do not have to be zero between two identical charges because they have no direction.*

4. A charge (Q_1 = 25.0 μC) is place at the point (1.00 m, -2.00 m) on an x-y plane. A second charge (Q_2 = 100 μC) is placed at the point (4.00 m, 2.00m). Assume these are the only two charges in the area. (A) Determine the direction of the electric force acting on charge Q_2. (B) Determine the magnitude of the electric force acting on charge Q_2. (C) Determine the x-component of the electric force acting on charge Q_2. (D) Determine the y-component of the electric force acting on charge Q_2. (E) Determine the direction of the electric force acting on charge Q_1. (F) Determine the magnitude of the electric force acting on charge Q_1. (G) Determine the x-component of the electric force acting on charge Q_1. (H) Determine the y-component of the electric force acting on charge Q_1.

   A. *First, draw a picture. As both charges are positive, the force will be repulsive. That makes the force on Q_2 move in the direction from Q_1 to Q_2.*



$$\vec{r} = \vec{P}_{End} - \vec{P}_{Start}$$

$$\vec{r} = \{(4.00\ m)\hat{\imath} + (2.00\ m)\hat{\jmath}\} - \{(1.00\ m)\hat{\imath} + (-2.00\ m)\hat{\jmath}\}$$

$$\vec{r} = (4.00\ m)\hat{\imath} + (2.00\ m)\hat{\jmath} - (1.00\ m)\hat{\imath} + (2.00\ m)\hat{\jmath}$$

$$\vec{r} = (3.00\ m)\hat{\imath} + (4.00\ m)\hat{\jmath}$$

$$\theta_2 = Tan^{-1}\left(\frac{y}{x}\right) = Tan^{-1}\left(\frac{4.00\,m}{3.00\,m}\right) = 53.1°$$

B. $r = \sqrt{x^2 + y^2} = \sqrt{(3.00\,m)^2 + (4.00\,m)^2} = 5.00\,m$

$$F_2 = k\frac{Q_1 Q_2}{r^2} = \left(9.00 \times 10^9\,\frac{N \cdot m^2}{C^2}\right)\frac{(25.0 \times 10^{-6}\,C)(100 \times 10^{-6}\,C)}{(5.00\,m)^2} = 0.900\,N$$

C. $F_{2x} = F_2 \cdot Cos(\theta) = (0.900\,N) \cdot \frac{3.00\,m}{5.00\,m} = 0.540\,N$

D. $F_{2y} = F_2 \cdot Sin(\theta) = (0.900\,N) \cdot \frac{4.00\,m}{5.00\,m} = 0.720\,N$

E. *By Newton's 3rd Law, F_1 must be equal in magnitude but opposite in direction to F_2. So must all of the components.*

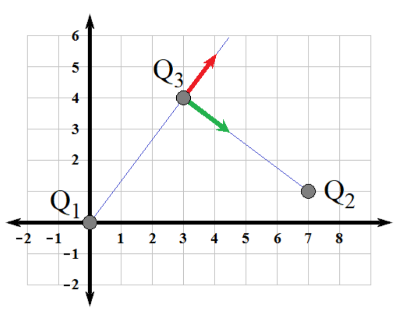
$$\theta_1 = \theta_2 - 180° = 53.1° - 180° = -126.9°$$

F. $F_1 = F_2 = 0.900\,N$

G. $F_{1x} = -F_{2x} = -0.540\,N$

H. $F_{1y} = -F_{2y} = -0.720\,N$

5. Three charges are placed on an xy-plane. A 5.00 μC charge (Q_1) is fixed at the origin. A second charge (Q_2 = -10.0 μC) is fixed at the point (7.00 m, 1.00 m). A third charge (Q_3 = 5.00 μC) is placed at the point (3.00 m, 4.00 m). Determine (A) the x-component, and (B) the y-component of the net force acting on charge Q_3.



$$\vec{r}_{13} = \vec{P}_3 - \vec{P}_1 = \{(3.00\,m)\hat{\imath} + (4.00\,m)\hat{\jmath}\} - \{(0)\hat{\imath} + (0)\hat{\jmath}\} = (3.00\,m)\hat{\imath} + (4.00\,m)\hat{\jmath}$$

$$r_{13} = \sqrt{x_{13}^2 + y_{13}^2} = \sqrt{(3.00\,m)^2 + (4.00\,m)^2} = 5.00\,m$$

$$\vec{r}_{23} = \vec{P}_3 - \vec{P}_2 = \{(3.00\,m)\hat{\imath} + (4.00\,m)\hat{\jmath}\} - \{(7.00\,m)\hat{\imath} + (1.00\,m)\hat{\jmath}\} = (-4.00\,m)\hat{\imath} + (3.00\,m)\hat{\jmath}$$

$$r_{23} = \sqrt{x_{23}^2 + y_{23}^2} = \sqrt{(4.00\,m)^2 + (3.00\,m)^2} = 5.00\,m$$

$$F_{13} = k\frac{Q_1 Q_3}{r_{13}^2} = \left(9.00 \times 10^9\,\frac{N \cdot m^2}{C^2}\right)\frac{(5.00 \times 10^{-6}\,C)(5.00 \times 10^{-6}\,C)}{(5.00\,m)^2} = 9.00\,mN$$

$$F_{23} = k\frac{Q_2 Q_3}{r_{23}^2} = \left(9.00 \times 10^9\,\frac{N \cdot m^2}{C^2}\right)\frac{(10.0 \times 10^{-6}\,C)(5.00 \times 10^{-6}\,C)}{(5.00\,m)^2} = 18.0\,mN$$

$$F_{3x} = F_{13} \cdot Cos(\theta_{13}) + F_{23} \cdot Cos(\theta_{23}) = (9.00\,mN) \cdot \left(\frac{3.00\,m}{5.00\,m}\right) + (18.0\,mN) \cdot \left(\frac{4.00\,m}{5.00\,m}\right) = 19.8\,mN$$

$$F_{3y} = F_{13} \cdot Sin(\theta_{13}) + F_{23} \cdot Sin(\theta_{23}) = (9.00\,mN) \cdot \left(\frac{4.00\,m}{5.00\,m}\right) + (18.0\,mN) \cdot \left(\frac{-3.00\,m}{5.00\,m}\right) = -3.6\,mN$$

6. Three identical conducting spheres are sitting in a lab, each on stand that is a perfect electric insulator. The first sphere has a net charge of $Q_1 = 55.0\ \mu C$. The second sphere has a net charge of $Q_2 = 33.0\ \mu C$, and the third sphere has a net charge of $Q_3 = 20.0\ \mu C$. The first two spheres are briefly held together and then separated. Then the second and third sphere are briefly held together (touching) and then separated. After this is done, determine the electric repulsion between the first and the third charge if they are placed 4.00 m apart.

   When two spheres are held together, their charges are free to mix. Half of the net charge ends up on each sphere.

   Sphere 1 & 2 touch: $Q_{New1} = \frac{(55.0\ \mu C)+(33.0\ \mu C)}{2} = 44.0\ \mu C$

   Sphere 2 & 3 touch: $Q_{New2} = \frac{(44.0\ \mu C)+(20.0\ \mu C)}{2} = 32.0\ \mu C$

   $$F_{13} = k\frac{Q_1Q_3}{r_{13}^2} = \left(9.00\ \times 10^9\ \frac{N \cdot m^2}{C^2}\right)\frac{(44.0\ \times\ 10^{-6}\ C)(32.0\ \times\ 10^{-6}\ C)}{(4.00\ m)^2} = 792\ mN$$

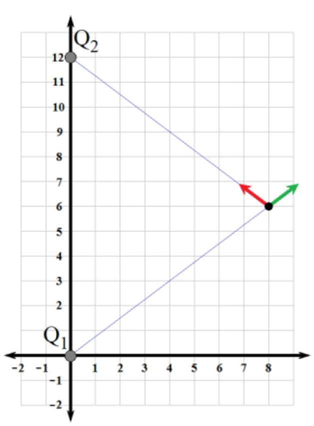
7. When a small object is placed in an electric field with magnitude 3.00 kN/C, it experiences a force of 51.0 nN. What is the charge on that object?

   $$F = qE \qquad q = \frac{F}{E} = \frac{51.0 \times 10^{-9}\ N}{3.00 \times 10^3\ \frac{N}{C}} = 17.0\ pC$$

8. How far from a 25.0 nC charge is the magnitude of its electric field 9.00 N/C?

   $$E = k\frac{Q}{r^2} \qquad r = \sqrt{k\frac{Q}{E}} = \sqrt{\left(9.00\ \times 10^9\ \frac{N \cdot m^2}{C^2}\right)\frac{(25.0 \times 10^{-9}\ C)}{\left(9.00\frac{N}{C}\right)}} = 5.00\ m$$

9. A charge ($Q_1 = 200$ nC) is placed at the origin. A second charge ($Q_2 = -300$ nC) is placed on the y-axis at y = 12.0 m. Assuming there are no other charges nearby, determine the magnitude of the net electric field at the point P (8.00 m, 6.00 m).



$$\vec{r}_1 = \vec{P} - \vec{P}_1 = \{(8.00\ m)\hat{\imath} + (6.00\ m)\hat{\jmath}\} - \{(0)\hat{\imath} + (0)\hat{\jmath}\} = (8.00\ m)\hat{\imath} + (6.00\ m)\hat{\jmath}$$

$$r_1 = \sqrt{x_1^2 + y_1^2} = \sqrt{(8.00\ m)^2 + (6.00\ m)^2} = 10.0\ m$$

$$\vec{r}_2 = \vec{P}_2 - \vec{P} = \{(0)\hat{\imath} + (12.00\ m)\hat{\jmath}\} - \{(8.00\ m)\hat{\imath} + (6.00\ m)\hat{\jmath}\} = (-8.00\ m)\hat{\imath} + (6.00\ m)\hat{\jmath}$$

$$r_2 = \sqrt{x_2^2 + y_2^2} = \sqrt{(8.00\ m)^2 + (6.00\ m)^2} = 10.0\ m$$

$$E_1 = k\frac{Q_1}{r_1^2} = \left(9.00\ \times 10^9\ \frac{N \cdot m^2}{C^2}\right)\frac{(200\ \times\ 10^{-9}\ C)}{(10.0\ m)^2} = 18.0\ N$$

$$E_2 = k\frac{Q_2}{r_2^2} = \left(9.00\ \times 10^9\ \frac{N \cdot m^2}{C^2}\right)\frac{(300\ \times\ 10^{-9}\ C)}{(10.0\ m)^2} = 27.0\ N$$

$$E_x = E_{1x} + E_{2x} = E_1 \cdot Cos(\theta_1) + E_2 \cdot Cos(\theta_2) = (18.0\ N) \cdot \frac{8.00\ m}{10.0\ m} + (27.0\ N)\frac{-8.00\ m}{10.0\ m} = 7.20\ N$$

$$E_y = E_{1y} + E_{2y} = E_1 \cdot Sin(\theta_1) + E_2 \cdot Sin(\theta_2) = (18.0\ N) \cdot \frac{6.00\ m}{10.0\ m} + (27.0\ N)\frac{6.00\ m}{10.0\ m} = 27.0\ N$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(7.20\ N)^2 + (27.0\ N)^2} = 27.9\ N$$

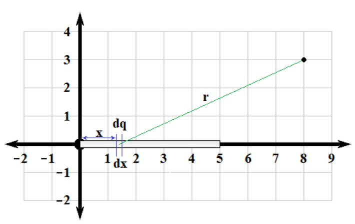
10. Determine (A) the magnitude of the acceleration, and (B) the direction of the acceleration of an electron that is placed in an electric field given by $\vec{E} = \left(4.00\frac{\mu N}{C}\right)\hat{\imath} + \left(3.00\frac{\mu N}{C}\right)\hat{\jmath}$.

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{\left(4.00\frac{\mu N}{C}\right)^2 + \left(3.00\frac{\mu N}{C}\right)^2} = 5.00\frac{\mu N}{C}$$

$$\theta = Tan^{-1}\left(\frac{E_y}{E_x}\right) = Tan^{-1}\left(\frac{3.00\frac{\mu N}{C}}{4.00\frac{\mu N}{C}}\right) = 36.9°$$

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m} = \frac{1.60\times10^{-19}\ C}{9.11\times10^{-31}\ kg}\left(5.00 \times 10^{-6}\frac{\mu N}{C}\angle 36.9°\right) = 878\frac{km}{s^2}\angle 36.9°$$

11. One end of a thin rod is placed at the origin, and it extends along the x-axis to x_1 = 5.00 cm. Set up the integrals to find (A) the x-component of the electric field and (B) the y-component of the electric field at the point P = (x_2, y_2) = (8.00 cm, 3.00 cm).



$$dq = \lambda dx = \frac{Q}{L}dx$$

$$dE = k\frac{dq}{r^2} = \frac{kQ}{L}\left(\frac{dx}{r^2}\right) = \frac{kQ}{L}\left(\frac{dx}{(0.08\ m - x)^2 + (0.03\ m)^2}\right)$$

$$dE_x = dE \cdot Cos(\theta) = dE \cdot \frac{(0.08\ m - x)}{\sqrt{(0.08\ m - x)^2 + (0.03\ m)^2}}$$

$$dE_x = \frac{kQ}{L}\left(\frac{dx}{(0.08\ m - x)^2 + (0.03\ m)^2}\right)\frac{(0.08\ m - x)}{\sqrt{(0.08\ m - x)^2 + (0.03\ m)^2}} = \frac{kQ}{L}\left(\frac{(0.08\ m - x)dx}{[(0.08\ m - x)^2 + (0.03\ m)^2]^{\frac{3}{2}}}\right)$$

$$E_x = \int_0^L dE_x = \frac{kQ}{L}\int_0^L\left(\frac{(0.08\ m - x)dx}{[(0.08\ m - x)^2 + (0.03\ m)^2]^{\frac{3}{2}}}\right)$$

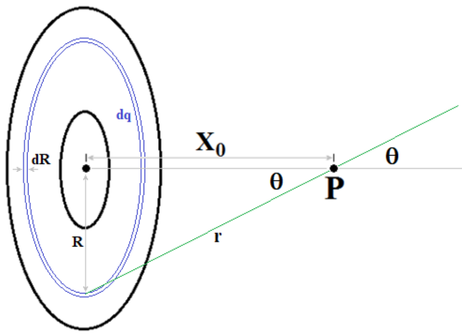
$$dE_y = dE \cdot Sin(\theta) = dE \cdot \frac{(0.03\ m)}{\sqrt{(0.08\ m - x)^2 + (0.03\ m)^2}}$$

$$dE_y = \frac{kQ}{L}\left(\frac{dx}{(0.08\ m - x)^2 + (0.03\ m)^2}\right)\frac{(0.03\ m)}{\sqrt{(0.08\ m - x)^2 + (0.03\ m)^2}} = \frac{kQ}{L}\left(\frac{(0.03\ m)dx}{[(0.08\ m - x)^2 + (0.03\ m)^2]^{\frac{3}{2}}}\right)$$

$$E_y = \int_0^L dE_y = \frac{kQ}{L}\int_0^L\left(\frac{(0.03\ m)dx}{[(0.08\ m - x)^2 + (0.03\ m)^2]^{\frac{3}{2}}}\right)$$

To get to a numeric answer, you could substitute "Z = (0.08 m-x)", "dZ = -dx", and "a = 0.03 m" and go look this up on an integral table. Also, L = 0.05 m.

12. A non-conducting annulus (the area between two concentric circles) with inner radius R_1 and outer radius R_2, has a charge Q uniformly distributed on one face. Determine the magnitude of the electric field at a point P, which is distance x_0 away from the center of the annulus along its rotational axis.



The transverse components of the electric field (y and z) will cancel out. This can be deduced by looking at the symmetry of the problem. So, $E_y = E_z = 0$. We need only find E_x as $E = E_x$.

This allows us to integrate using a thin circular ring as all the parts of it produce the same value for dE_x. The ring (outlined in blue) has charge dq and thickness dR.

$$\sigma = \frac{Q}{A} = \frac{Q}{\pi R_2^2 - \pi R_1^2} = \frac{Q}{\pi(R_2^2 - R_1^2)}$$

$$dq = \sigma dA = \sigma \cdot 2\pi R dR = \frac{2\pi Q R \cdot dR}{\pi(R_2^2 - R_1^2)} = \frac{2Q}{(R_2^2 - R_1^2)} R \cdot dR$$

$$E = E_x = \int_{R_1}^{R_2} dE_x = \int_{R_1}^{R_2} dE \cdot Cos(\theta) = \int_{R_1}^{R_2} dE\,\frac{x_0}{r} = \int_{R_1}^{R_2} \left(k\frac{dq}{r^2}\right)\frac{x_0}{r} =$$

$$E = \frac{2Q}{(R_2^2 - R_1^2)} \int_{R_1}^{R_2} \left(k\frac{R \cdot dR}{r^2}\right)\frac{x_0}{r} = \frac{2Qkx_0}{(R_2^2 - R_1^2)} \int_{R_1}^{R_2} \left(\frac{R \cdot dR}{r^3}\right)$$

$$E = \frac{2Qkx_0}{(R_2^2 - R_1^2)} \int_{R_1}^{R_2} \left(\frac{R \cdot dR}{[R^2 + x_0^2]^{\frac{3}{2}}}\right)$$

Let $U = [R^2 + x_0^2]^{-\frac{1}{2}}$, *then* $dU = -\frac{1}{2}[R^2 + x_0^2]^{-\frac{3}{2}} \cdot 2R \cdot dR = \frac{-R \cdot dR}{[R^2 + x_0^2]^{\frac{3}{2}}}$

$$E = \frac{2Qkx_0}{(R_2^2 - R_1^2)} \int_{R_1}^{R_2} \left(\frac{R \cdot dR}{[R^2 + x_0^2]^{\frac{3}{2}}}\right) = \frac{-2Qkx_0}{(R_2^2 - R_1^2)} \int_{R_1}^{R_2} dU = \frac{-2Qkx_0}{(R_2^2 - R_1^2)} [U]_{R_1}^{R_2}$$

$$E = \frac{-2Qkx_0}{(R_2^2 - R_1^2)} \left[\frac{1}{\sqrt{R^2 + x_0^2}}\right]_{R_1}^{R_2} = \frac{2Qkx_0}{(R_2^2 - R_1^2)} \left[\frac{1}{\sqrt{R_1^2 + x_0^2}} - \frac{1}{\sqrt{R_2^2 + x_0^2}}\right]$$