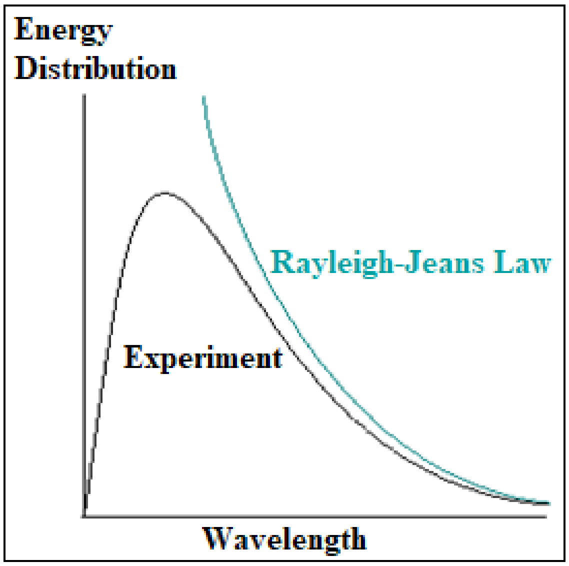
Part 9: Quantum Mechanics

College Physics (Openstax): Chapters 29 & 30
Physics: Principles with Applications (Giancoli): Chapters 27 & 28

The "Ultraviolet Catastrophe"

Prior to 1900 there was vast disagreement between experimental results and the theoretical predictions for the black body spectrum, particularly at the peak of the experimental curve (the ultraviolet region).



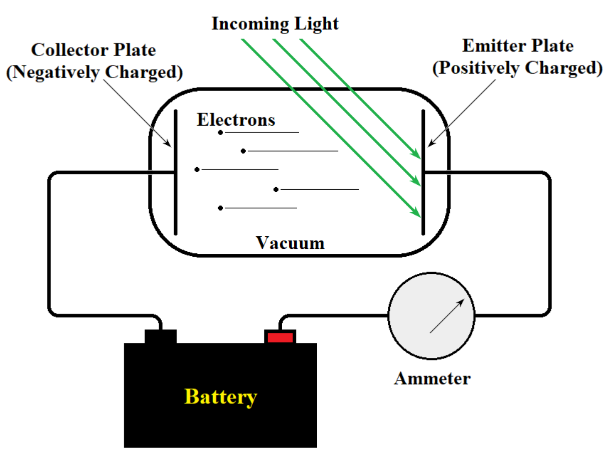
- In 1900, Max Planck discovered that he could derive this curve, but to do so he had to assume that light was emitted in discrete bundles of energy (quantized).
- $E = hf$ or $E = \hbar\omega$, where $h = 6.63\times10^{-34} J\cdot s$

Reduced Planck's Constant: $\hbar = \dfrac{h}{2\pi}$

Planck assumed this quantization was created by the emission process.

Photoelectric Effect

- Light striking a metal plate can liberate electrons. This is called the photoelectric effect and the electrons are called photoelectrons.



- Light striking the emitter frees photoelectrons.
- As the collector is negative and the emitter is positive, the electrons must have enough kinetic energy to overcome the potential V to reach the collector plate allowing current to flow. (*Note: V is backwards in the book*)
- The voltage is slowly increased until the current stops. This voltage then equates to maximum kinetic energy of the liberated photoelectrons. (eV=1/2mv2)

If

- light is a wave…
  - KE_{Max} should increase with intensity as more energy is deposited
  - Changing the frequency should have no effect.
- Increasing intensity had no effect on KE_{Max} but increased the current.
- KE_{Max} increased with frequency.
- This makes no sense if light is a wave!
- Einstein (1905) explained that these results were consistent with a particle description of light (photons) rather than a wave description.

$$KE_{Max} = hf - W_0$$

  - "hf" is the photon energy.
  - "W_0", the "work function" is the energy needed to break the electrons bonds.
- Increasing the intensity sends more photons, liberating more electrons, increasing the current.
- Increasing the frequency increases the photon energy providing more KE to the electrons.

So light is clearly a particle!
But we saw evidence previously that showed light is clearly a wave!
But these different phenomena are mutually exclusive!

Sometimes light is a particle and sometimes light is a wave!
Light travels as a wave, but it interacts as a particle!

Example: Ultraviolet light is responsible for sun tanning. Find the wavelength of an ultraviolet photon of energy 6.40×10^{-19}J (4.00 eV).

$$E = hf = h\frac{c}{\lambda} \qquad \lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\,J\cdot s)(3.0\times10^{8}\,m/s)}{6.40\times10^{-19}\,J} = 311nm$$

Example: Ultraviolet light with a frequency of 3.00×10^{15}Hz strikes a metal surface and ejects electrons that have a maximum kinetic energy of 6.1 eV. What is the work function of the metal?

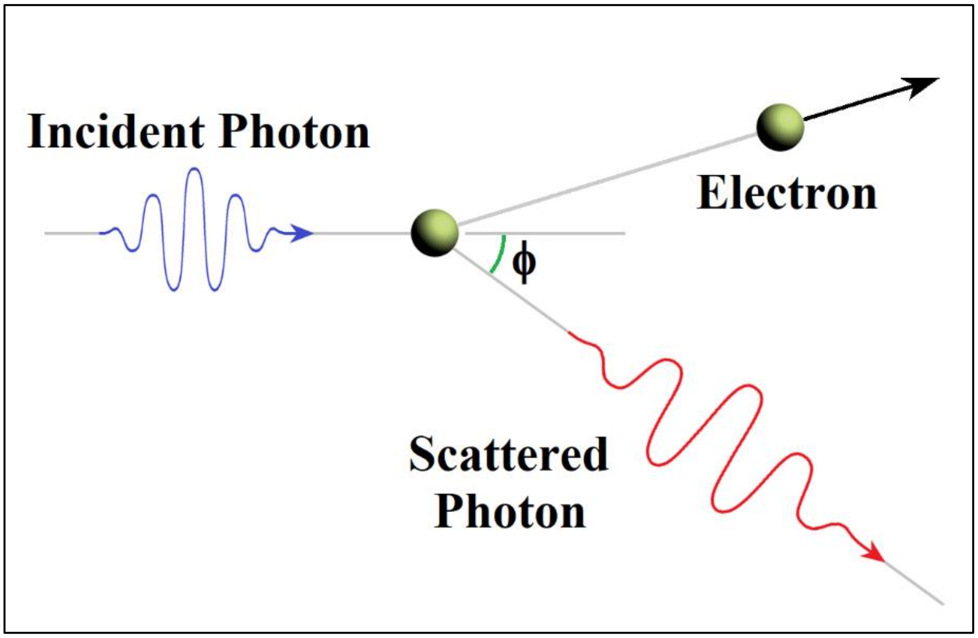
$$KE_{Max} = hf - W_0 \qquad W_0 = hf - KE_{Max}$$

$$hf = (6.63\times10^{-34}\,J\cdot s)(3.00\times10^{15}\,Hz)\frac{1eV}{1.60\times10^{-19}\,J} = 12.4eV$$

$$W_0 = hf - KE_{Max} = 12.4eV - 6.1eV = 6.3eV$$

Compton Effect

- Compton discovered that photons can collide with stationary electrons.
- The photon "deflects" as a lower energy photon, imparting energy to the electron in the process.



But how is momentum conserved when the photon has no mass?

While the photon has no mass, it does have momentum!

$$E^2 = p^2c^2 + m_0^2c^4 = p^2c^2 \qquad p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

- Using conservation of energy and momentum (elastic collision) along with E=hf and p=h/λ, Compton derived an equation that matched the experimental results.

$$\lambda' - \lambda = \frac{h}{mc}(1 - Cos\phi)$$

  - λ' = wavelength of scattered photon.
  - λ = wavelength of incident photon
  - h/mc = "Compton wavelength" of an electron
  - ϕ = photon scattering angle

Example: The microwaves in a microwave oven have a wavelength of 0.13 m. What is their momentum?

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\,J\cdot s}{0.13\text{m}} = 5.1\times10^{-33}\,\frac{\text{kg}\cdot\text{m}}{\text{s}}$$

Example: A photon of wavelength 4.1 pm strikes an electron at rest. Determine the kinetic energy gained by the electron if the scattered photon is detected at an angle of 135°.

The kinetic energy gained by the electron is the difference of the energy of the incoming photon and the energy of the outgoing photon. Photon energies are dependent upon frequency/wavelength.

$$\lambda' - \lambda = \frac{h}{mc}(1 - Cos\phi) \qquad \lambda' = \lambda + \frac{h}{mc}(1 - Cos\phi)$$

$$\lambda' = 4.10\,pm + \frac{6.63\times10^{-34}\,J\cdot s}{(9.11\times10^{-31}kg)(3.00\times10^{8}\,m/s)}\{1 - Cos(135°)\} = 8.24\,pm$$

$$E_{incident} = E_{scattered} + KE_{electron}$$

$$KE = hf_i - hf_s = hc\left\{\frac{1}{\lambda_i} - \frac{1}{\lambda_s}\right\}$$

$$KE = (6.63\times10^{-34}\,J\cdot s)(3.00\times10^{8}\,m/s)\left\{\frac{1}{4.1\,pm} - \frac{1}{8.24\,pm}\right\} = 2.4\times10^{-14}\,J$$

Louis de Broglie

- His doctoral dissertation proposed that if light was both a particle and a wave, perhaps other things that we think of as particles are also waves.
- His examiners had no idea what to do with this, and so passed it on to Einstein.
- After Einstein's endorsement, de Broglie was granted his doctorate.

$$\lambda = \frac{h}{p}$$

- λ= electron's wavelength
- h = Planck's constant
- p = relativistic momentum

Do particles like electrons make interference patterns? YES!

What happens when you fire one electron at a time through a double slit experiment? One would think that the electron can go through only one slit or the other, preventing interference from occurring.

But NO! An interference pattern still forms once enough data has been collected!

How can this be?!?

We don't know. But it is!

We'll come back to this.

Rutherford Scattering: The Nuclear Atom

- Prior to 1911, atoms were thought to look like "plum pudding" with electrons floating in a positive "paste".
- In 1911, Rutherford fired alpha particles, which we now know are helium nuclei, at a thin gold film. As these were much more massive than electrons, it was assumed that nothing would deflect them.
- **While most passed through, a small number were deflected backwards.**
- Rutherford concluded that the positive charge was concentrated in a dense object in the center of the atom. This was dubbed the **Nuclear Atom** with a dense positive nucleus (r≈10-15m) surrounded by orbiting electrons (r≈10-15m).
- There was a problem with this. Accelerating electrons emit EM radiation. Orbiting electrons (under centripetal acceleration) should emit photons causing their orbits to decay and allowing them to spiral into the nucleus.

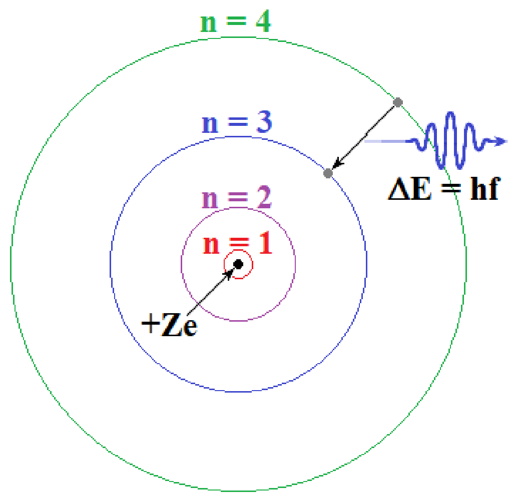
Emission Line Spectra

- It was also not understood why atoms only emitted certain frequencies. When light was sent through a dispersive media (which separated the colors) these showed up as colored lines (hence "line spectra").
- Patterns were found in the hydrogen spectra, but lacked an explanation.
  - R = Rydberg Constant = $1.097\text{x}10^7\ \text{m}^{-1}$
  - Lyman Series: $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \quad n = 2,3,4...$
  - Balmer Series: $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad n = 3,4,5...$
  - Paschen Series: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right) \quad n = 4,5,6...$

These series are useful, but they don't explain why these particular frequencies are emitted by hydrogen atoms.

The Bohr Model (1913)

- Since photon energy is quantized, these spectra mean specific energy transitions must occur inside an atom. $E_{init} - E_{final} = hf$
- The atom's energy must be the sum of electron's kinetic and potential energy.
- Electrons must have specific "stationary orbits", and transitions from one to the next must be accompanied by the release of a single photon.



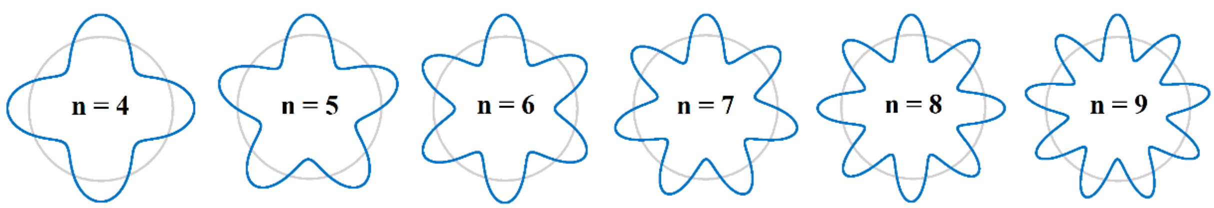
- $E = KE + EPE = \frac{1}{2} m_e v^2 - k\frac{e^2}{r}$
- $F = \frac{mv^2}{r} = k\frac{e^2}{r^2} \qquad mv^2 = k\frac{e^2}{r}$
- $E = \frac{1}{2} m_e v^2 - k\frac{e^2}{r} = \frac{ke^2}{2r} - \frac{ke^2}{r} = -\frac{ke^2}{2r}$
- If the energy is a function of r, and the energy is quantized, then r must also be quantized. But how?

- Bohr assumed that angular momentum must be quantized.

$$L = I\omega = \left(m_e r^2\right)\left(\frac{v}{r}\right) = m_e v r$$

$$L_n = m_e v_n r_n = \frac{nh}{2\pi} \qquad n = 1,2,3\ldots$$

- de Broglie (1923) explained it better. The electron is a wave, and a standing wave must "fit" inside the container (only certain wavelengths are allowed).



$$2\pi r = n\lambda \quad n = 1,2,3\ldots$$

$$2\pi r = n\lambda = \frac{nh}{p} = \frac{nh}{mv}$$

$$mvr = \frac{nh}{2\pi} \qquad v = \frac{nh}{2\pi m r}$$

- Start with the Coulomb force being the centripetal force (multiply through by r):

$$mv^2 = k\frac{e^2}{r}$$

- Insert $v = \frac{nh}{2\pi m r}$ (from our quantization condition):

$$m\frac{n^2h^2}{4\pi^2 m^2 r^2} = k\frac{e^2}{r}$$

- Solve for r:

$$r = \frac{n^2h^2}{4\pi^2ke^2m} = (5.29\times10^{-11}m)n^2$$

  - r = $5.29\text{x}10^{-11}$m is called the "Bohr Radius" (the size of a hydrogen atom)
- Plug the Bohr Radius into the equation to calculate the energy of the hydrogen atom:

$$E = -\frac{ke^2}{2r} = \frac{2\pi^2k^2e^4m}{n^2h^2} = \frac{-13.6eV}{n^2}$$

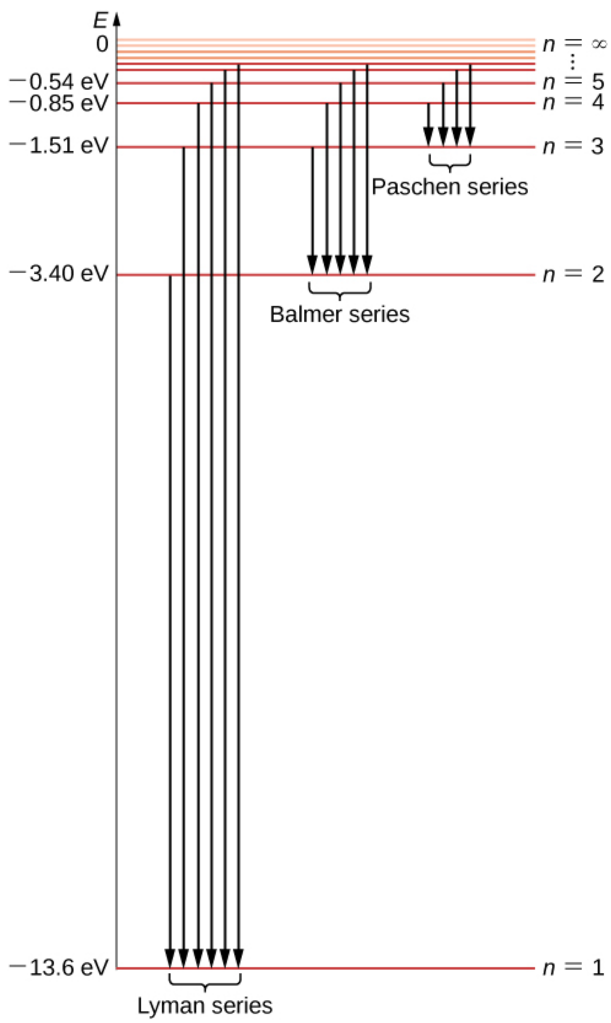
- Use the difference in two energy levels to calculate the wavelength of an emitted photon:

$$\frac{1}{\lambda} = \frac{f}{c} = \frac{\Delta E}{ch} = \frac{2\pi^2k^2e^4m}{ch^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

- The leading coefficient produces the Rydberg constant:

$$\frac{2\pi^2k^2e^4m}{ch^3} = 1.097\times10^7\,\text{m}^{-1} = \text{R}$$

The Bohr model explains the entire emission spectrum for Hydrogen!



Erwin Schrodinger

- "If an electron is a wave, what is the wave equation?"
- The Schrodinger Equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\psi$$

Max Born

- What is Ψ? What is a wave function?
- Max Born determined that $|\Psi|^2$ was the probability distribution (i.e. the probability that the particle will be found at a given location)
- As Ψ is complex, $|\Psi|^2 = \Psi^*\Psi = <\Psi|\Psi>$, where Ψ^*= complex conjugate
- This makes quantum mechanics probabilistic in nature.

All of the determinism of Newtonian mechanics arises from the underlying randomness of quantum mechanics!

"God does not play dice!" – Albert Einstein

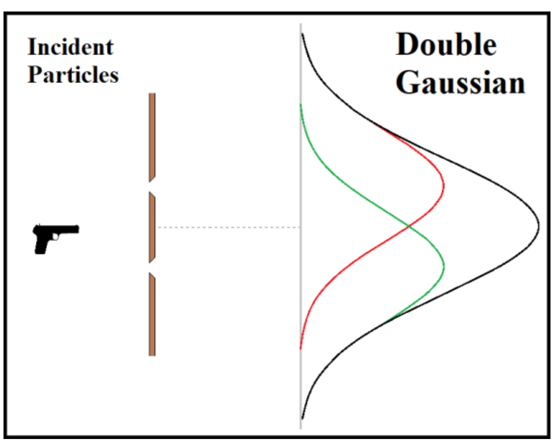
"Stop telling God what to do!" – Neils Bohr

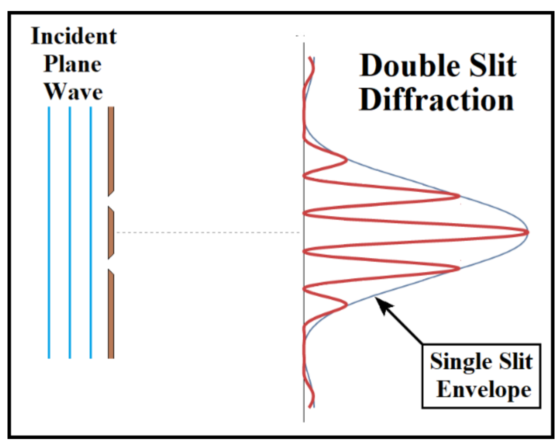
***Strange fact**: Max Born has a Bacon number of 2, as his granddaughter, Olivia Newton-John was in "She's having a baby" with Kevin Bacon.*

This means Einstein, Bohr, Schrodinger, and all the other members of the Solvay Conference (other than Born) have a Bacon number of 3.

Interpreting the Double Slit Experiment

What does Schrodinger's equation imply is happening in the double slit experiment?

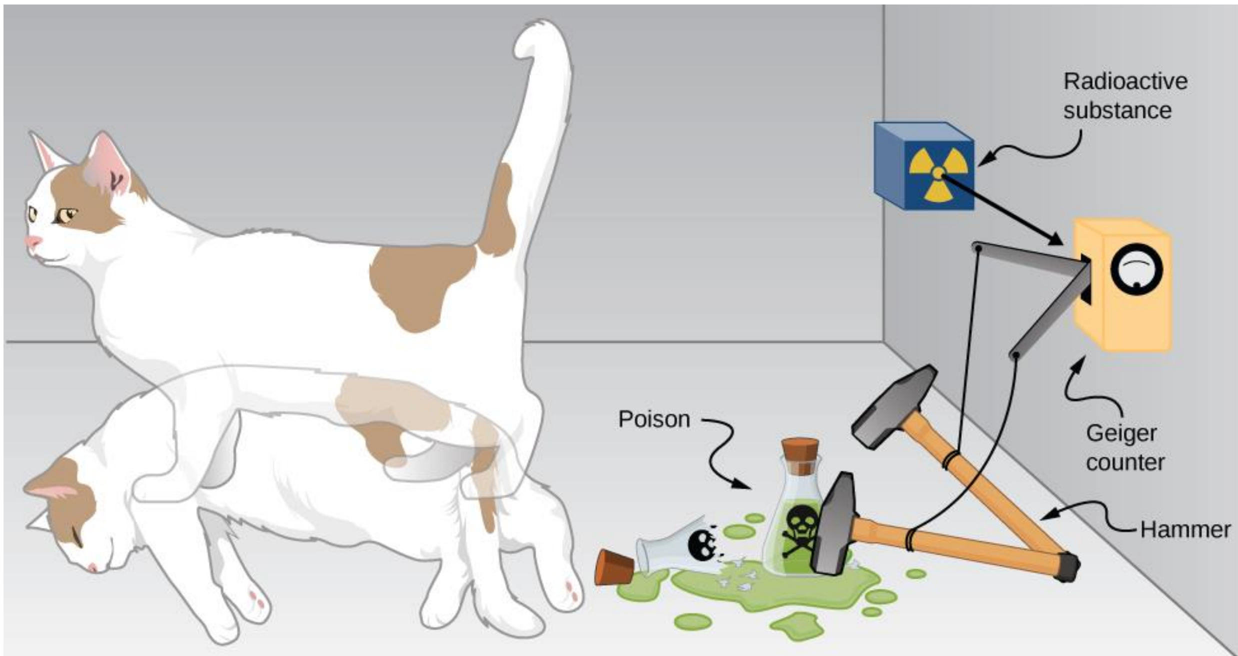




- Particles move as waves (described by the wave function) until they hit the detection screen, when they "magically" become a particle.

- If you detect which slit the particle goes through, that "collapses" the wave function, causing it to become a particle (destroying the interference).
- This implies that observation is sufficient to collapse a wave function.

Schrodinger's Cat



- If we take a radioactive atom and wait one half life, then it has a 50% of decaying. But this is a quantum particle! $\rightarrow$ Superposition of states!
- If we take that radioactive atom and enclose it in a detector to detect its decay, then that detector must go into a superposition of states.
- If the detector is rigged to break a vial of poison, then the vial goes into a superposition of states (both broken and not broken).
- And if we put a cat in the box, then the cat must be in a superposition of states (alive and dead), until I open the box and look.
- This is absurd. Therefore quantum mechanics must be wrong!

We have now put objects as big as 30 μm into a superposition of states.

Interpretations of Quantum Mechanics

Niels Bohr: "Everything we call real is made of things that cannot be regarded as real"

Neils Bohr: "If quantum mechanics hasn't profoundly shocked you, you haven't understood it yet."

- **The Hidden Variable Interpretation** (Einstein)
    - Quantum mechanics only appears probabilistic because there are properties in particles (of which we are unaware) that determine where it lands.
    - This has been shown to be wrong.
- **The Pilot Wave Interpretation** (Bohm)
    - The probability wave and particle are two separate entities, and the wave guides the particle to its final destination. (also proven wrong)
    - This has also been shown to be wrong.
- **The Copenhagen Interpretation** (Bohr, Heisenberg)
    - Only things that can be observed (measured) can be considered real. As wave functions cannot be detected, we must assume there is no underlying reality.

- **The Many Worlds Interpretation** (Everett)
    - The probability wave does not decohere/collapse. Rather each possible outcome occurs in its own a separate universe, constantly splitting away from others.

I don't think we've got it yet…

Heisenberg Uncertainty

- The relationship between p and x in the wave functions produced by Schrodinger's equation (as well as between E and t), make them "complimentary variables"

$$\Delta x \cdot \Delta p_x \geq \frac{\hbar}{2} \qquad \Delta t \cdot \Delta E \geq \frac{\hbar}{2}$$

Note: Your book does what lazy physicists do and uses ħ instead of ħ/2.

- At first it was thought that this was an issue of measurement, but this is not correct. This is a property of the particles (waves) themselves.

Example: Single slit interference: Minima at dSinθ = λ. (Width of central peak)

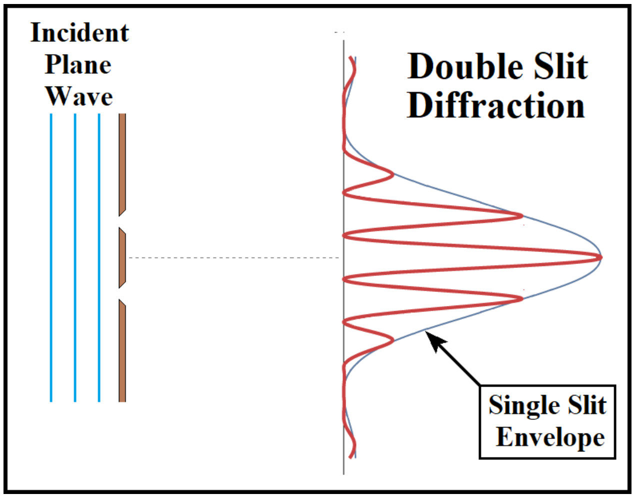
$$d = \Delta y \qquad Sin\theta \approx Tan\theta = \frac{\Delta p_y}{p} \qquad \lambda = \frac{h}{p}$$

$$dSin\theta = \lambda \quad \rightarrow \quad (\Delta y)\left(\frac{\Delta p_y}{p}\right) = \frac{h}{p} \quad \rightarrow \quad \Delta y \cdot \Delta p_y = h \geq \frac{\hbar}{2}$$

- $\Delta t \cdot \Delta E \geq \frac{\hbar}{2}$ implies that energy (ΔE) can be borrowed for short time intervals (Δt) to create (virtual) particles.

Weirdness

Richard Feynman: "If you think you understand quantum mechanics, you don't understand quantum mechanics. "



- **"Quantum Eraser"**
- Start with the double slit experiment.
- Put a vertical polarizer behind one slit, and a horizontal polarizer behind the other slit. Then use a screen sensitive to polarization.
- The interference patter is replaced with a Gaussian distribution.
- However, if you slide a polarizer with the transmission axis at 45° in front of the screen, you destroy the path information and the interference returns.

Extreme Weirdness

The Delayed Choice Quantum Eraser