Part 8: Special Relativity

College Physics (Openstax): Chapter 28
Physics: Principles with Applications (Giancoli): Chapter 26

The year 1900

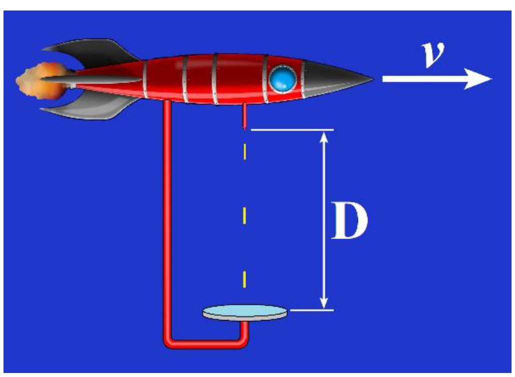
- Prior to 1900, there was a heated debate as to if we'd reached the end of physics..
- Between Newton's laws of motion, thermodynamics, and Maxwell's equations we could model almost all known phenomena.
- There were just a few minor details that no one could figure out, and it was expected that these would soon be solved
  - Why hadn't the sun burned out? If the sun was fueled by the strongest chemical reactions it would burn out in 6000 years
  - Why did the planet Mercury have a 3 degree precession? Elliptical planetary motions were well understood, but Mercury's ellipse rotated forward by 3° each orbit.
  - "The ultraviolet catastrophe". Calculation of black body radiation had a singularity in the ultraviolet region that was not there in experimental data.

Special Relativity (Introduction)

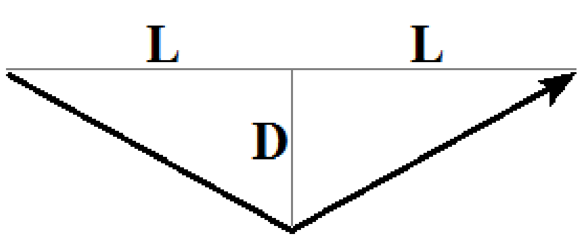
- A person is walking at exactly 3 m/s (relative to the train) from the back of the train to the front. The train is moving at exactly 30 m/s (relative to a stationary observer). How fast is the person moving as measured by the observer?
  - $V_{PO} = V_{PT} + V_{TO} = 3m/s + 30m/s = 33m/s$ Close → But technically wrong!
  - $V_{PO} = 32.99999999999966m/s$
    - This differs from 33 m/s by less than a thousandth of the width of an atom each second.
- A spaceship is travelling at 0.500c (half the speed of light in a vacuum) relative to a stationary observer. The pilot measures the speed of light (in the direction of his motion). The stationary observer uses the same experiment to measure the speed of light. As the speed of light is constant for all observers (Maxwell's equations), both measure the speed of light to be c = 3.00×10^8 m/s. How can this be?
  - The only possible way is that the two observers measure both time and distances differently!

- 1905: Einstein's Miraculous Year
  - He produced three papers worthy of a Nobel Prize: One on Brownian motion, one on the photoelectric effect, and one on special relativity.
  - He was 26.
- Special relativity deals with how to transform coordinates from one inertial reference frame { S = (x, y, z, t) } to a second inertial reference frame { S' = (x', y', z', t') }
  - **Inertial Reference Frame** – A reference frame where Newton's laws of motion are valid. Any reference frame moving with a constant velocity relative to an inertial reference frame is also an inertial reference frame.
  - Formerly, Galilean transformations were used. For a velocity (v) in the positive x-direction these are simply: x' = x-vt, y' = y, z' = z, and t'=t
    - This doesn't work when velocities approach the speed of light in vacuum.
- Einstein used 2 postulates to derive special relativity.
  - The laws of physics are the same in every inertial reference frame.
  - The speed of light in vacuum is the same in every inertial reference frame.

Time Dilation



- From the viewpoint of the pilot, the light pulses travel straight down and back.
- $2D = c\Delta t'$
- $D = \frac{c\Delta t'}{2}$



- From the viewpoint of a stationary observer watching the ship, the light pulses travel at an angle.
- $2L = v\Delta t$
- $L = \frac{v\Delta t}{2}$

$$c\Delta t = 2\sqrt{L^2 + D^2} \qquad c^2\Delta t^2 = 4(L^2 + D^2) \qquad c^2\Delta t^2 = 4\left[\left(\frac{v\Delta t}{2}\right)^2 + \left(\frac{c\Delta t'}{2}\right)^2\right]$$

$$c^2\Delta t^2 = v^2\Delta t^2 + c^2\Delta t'^2 \qquad \left(c^2 - v^2\right)\Delta t^2 = c^2\Delta t'^2 \qquad \left(1 - \frac{v^2}{c^2}\right)\Delta t^2 = \Delta t'^2$$

$$\sqrt{1 - \frac{v^2}{c^2}}\Delta t = \Delta t' \qquad \Delta t = \gamma \Delta t' \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \geq 1$$

Example: A law enforcement officer in a space ship turns on a red flashing light on top of the ship and sees it generate a flash every 1.50 s. A person on Earth measures the time between flashes at 2.50 s. How fast is the ship moving relative to Earth?

$$\Delta t = \gamma \Delta t' \qquad \gamma = \frac{\Delta t}{\Delta t'} = \frac{2.50s}{1.50s} = 1.666...$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad 1 - v^2/c^2 = \frac{1}{\gamma^2} \qquad v = c\sqrt{1 - \frac{1}{\gamma^2}}$$

$$v = c\sqrt{1 - \frac{1}{\gamma^2}} = c\sqrt{1 - \frac{1}{(1.66...)^2}} = 0.800c = 2.40 \times 10^8 m/s$$

*Note that if you had an identical twin that travelled on a ship moving 0.8c relative to you for ten years (your time), they would only experience (and age) six years, becoming four years younger than you. This is known as the "**Twin Paradox**".*

Evidence for Time Dilation

- Einstein used calculated the effects of special relativity on the orbit of the planet Mercury, which moves very fast when closest to the sun. He determined it should precess by 3 degrees.
- Muons created in cosmic ray collisions in the upper atmosphere have a lifetime (at rest) of about 2.2 μs, which is insufficient for them to reach the surface of the Earth when travelling close to c. Yet, we measure 1 muon per cm2 per min on the Earth's surface. This is consistent with their lifetime increasing due to time dilation.
- In 1971, two atomic clocks (good to the nanosecond) were synchronized. One was placed on a jet that took off. When it returned the clocks were out of sync and the difference was predicted correctly by special relativity.
- The first global positioning system failed to work as the clocks on the satellites quickly got out of sync. Newer versions work as the effects of special relativity and general relativity are accounted for.
- For the last few years, collisions have been occurring at a rate of one every 25 ns in the LHC. No events have been found to violate special relativity.

Length Contraction

- A ship travels at velocity v relative to Earth to a planet that is a distance L away from Earth (as measured from Earth).

$$v = \frac{L}{\Delta t}$$

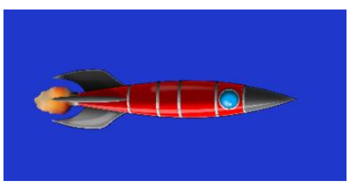
- The ship sees the star move towards him at a velocity v and sees it as a distance L' away.

$$v = \frac{L'}{\Delta t'}$$

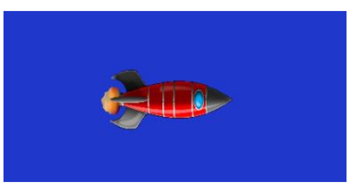
- $v = \frac{L'}{\Delta t'} = \frac{L}{\Delta t}$ $\qquad L' = \frac{\Delta t'}{\Delta t} L = \frac{1}{\gamma} L$

- We use the subscript "0" to denote the rest frame. Hence:
  - Length Contraction: $L = \frac{1}{\gamma} L_0$
  - Time Dilation: $t = \gamma t_0$

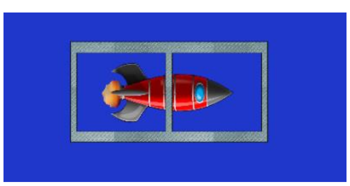
Example: Let's say that our ship is moving at 0.866c



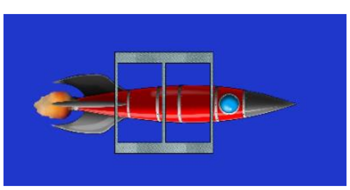
- Let's say that our ship is moving at 0.866c
- $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1-(0.866)^2}} = \frac{1}{0.500} = 2.00$



- If L0=50.0m, then L=L0/γ=25.0m



- Theoretically, we could take a picture of the 50.0m long ship in a 40.0m enclosure.



- From the pilot's point of view it's a little different. He sees the enclosure as 20.0m long. (L= L0/γ=40.0m/2)
- The stationary observer sees the enclosure doors shut simultaneously. The pilot doesn't. **Simultaneity is relative**.

As position and time in one frame become a mixture of position and time from the other frame, these are no longer separate entities $\rightarrow$ ***Spacetime!***

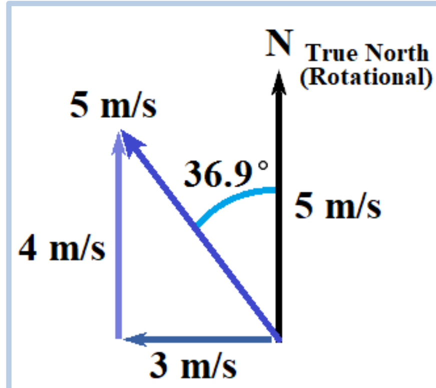
Coordinate Rotation

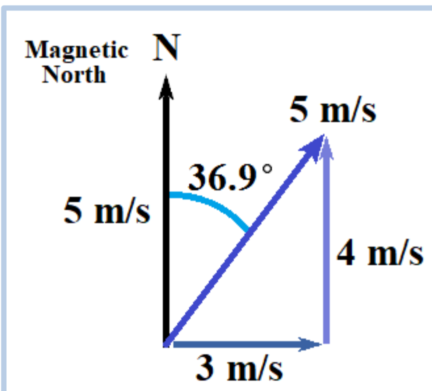
While on an expedition to Greenland, your group meets up with another group. As you are both heading north, you agree to travel together for safety, but the next day your dog sleds head off in different directions.

In your point of view, their north is a linear combination of your north and your west.

They are moving 4 m/s in your north direction and 3 m/s in your west direction.

You are moving 5 m/s to your north.





In their point of view, your north is a linear combination of their north and their west.

You are moving 4 m/s in their north direction, and 3 m/s in their east direction.

They are moving 5 m/s to their north.

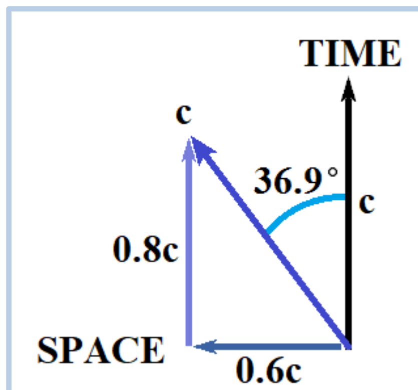
The only difference between these points of view is that one set of coordinate axes is rotated with respect to the other.

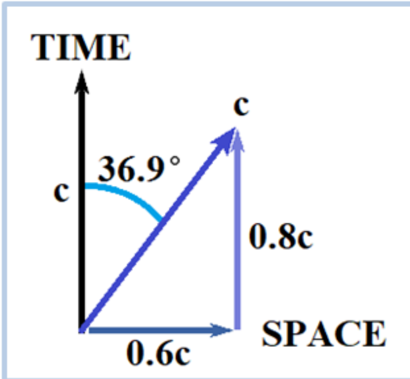
Special Relativity as a Coordinate Rotation

Let's consider a spaceship moving at 0.6c relative to you, a stationary observer.

In your point of view, their time is a linear combination of your time and your space.

In your view, for every 5 seconds that pass for you, only 4 seconds pass for them as they move away from you at 0.6 c. Their ship seems half as long to you as it does to them.

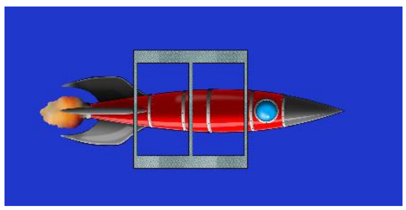


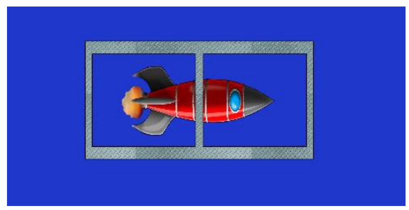


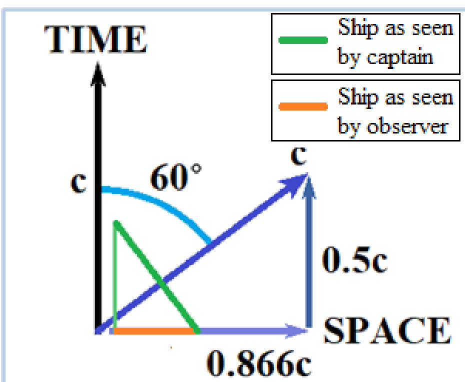
In their point of view, your time is a linear combination of their time and their space.

In their view, for every 5 seconds that pass for them, only 4 seconds pass for you as you move away from you at 0.6 c. you seem half as wide to them as you do appear to those standing near you.

The only difference between these points of view is that one set of coordinate axes has been rotated between time and space rather than just through space!
Let's revisit the 50.0 m long ship travelling at 0.866c relative to a 40.0 m long enclosure as it moves through.

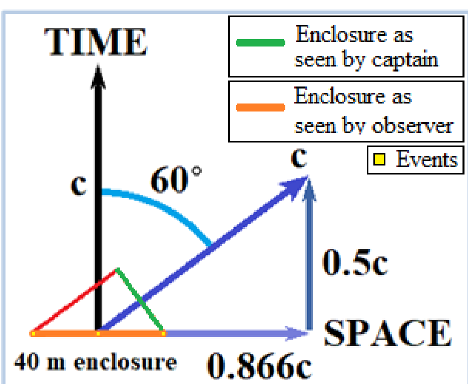






The captain of the ship sees a 50 m long ship.

The stationary observer sees the ship as 25 m long.



The stationary observer sees 3 simultaneous events happen along a 40 m enclosure (the front doors closing, the back doors closing, and the picture taken).

The captain of the ship sees 3 events separated in time over a 20 m long enclosure.

Relativistic Momentum and Energy

- These coordinate transformations at high velocities also mean that momentum and energy are no longer conserved. …unless we redefine them slightly.
    - $p = \gamma m_0 v$
    - $E = \gamma m_0 c^2$
- Relativistic Mass: $m = \gamma m_0$ …then $p = mv$
- Doing a little algebra

$$E^2 = \gamma^2 m_0^2 c^4 = \frac{1}{1-\frac{v^2}{c^2}} m_0^2 c^4 = \frac{c^2}{c^2 - v^2} m_0^2 c^4 = \left\{ \frac{c^2 - v^2}{c^2 - v^2} + \frac{v^2}{c^2 - v^2} \right\} m_0^2 c^4$$

$$E^2 = m_0^2 c^4 + \frac{c^2}{c^2 - v^2} m_0^2 v^2 c^2 = m_0^2 c^4 + \gamma^2 m_0^2 v^2 c^2 = m_0^2 c^4 + p^2 c^2$$

- Einstein Energy Momentum Relationship: $E^2 = m_0^2 c^4 + p^2 c^2$

Rest Energy

- The Einstein energy equation implies that mass is energy, and a massive object at rest still has a significant amount of energy.

$$E^2 = m_0^2 c^4 + p^2 c^2 \quad \rightarrow \quad E = m_0 c^2$$

- The united states has roughly 3,300 nuclear warheads with an average yield (*we're guessing here*) on the order of 1 megaton (total 3.3 gigatons)
    - One ton of TNT releases 4.184×109 J of energy
    - 3.3 Gtons = (4.184×109 J)(3.3×109) = 1.4×1019 J
    - m = 1.4×1019 J/c2 =153.4 kg (equivalent to 338 lbs)
- At the LHC, the protons will have total energy equal to 7000 times the mass of a proton. When a collision occurs some of this energy is transformed to create new particles (*such as the Higgs Boson, which has a mass 133 times larger than a proton)*
- Kinetic energy = Total Energy – Rest Energy

$$E = \gamma m_0 c^2 \qquad KE = \gamma m_0 c^2 - m_0 c^2 = (\gamma - 1) m_0 c^2$$

Low Energy Approximation $m_0^2c^4 >> p^2c^2$

$$E^2 = m_0^2c^4 + p^2c^2$$

$$E^2 - m_0^2c^4 = p^2c^2$$

$$(E + m_0c^2)(E - m_0c^2) = p^2c^2$$

$$(2m_0c^2)(KE) = p^2c^2$$

$$KE = \frac{p^2c^2}{2m_0c^2} = \frac{p^2}{2m_0} = \frac{m_0^2v^2}{2m_0} = \frac{1}{2}m_0v^2$$

At low energy, Einstein's relationship returns the kinetic energy

we used in Newtonian mechanics!

Note: A proton (mp = 938 MeV/c2) is composed of 2 up quarks (1.7 MeV/c2 < mu < 3.1 MeV/c2) and 1 down quark (4.1 MeV/c2 < md < 5.7 MeV/c2). Approximately 1% of a proton mass comes from its constituent quarks. Most of the mass comes from the binding energy of gluons.

Example: At what speed is the magnitude of the relativistic momentum of a particle three times the magnitude of the non-relativistic momentum?

$$p = \gamma m_0 v = 3m_0 v \qquad \gamma = 3 \qquad v = c\sqrt{1 - \frac{1}{\gamma^2}} = 0.94\text{c}$$

Example: A nuclear reactor generates 3.00 GW of power. In one year what is the change in the mass of the nuclear fuel rods?

$$E = Pt = \left(3.00\times10^9\ \frac{J}{s}\right)(1\ year)\left(\frac{365.25\ days}{1\ year}\right)\left(\frac{24\ hours}{1\ day}\right)\left(\frac{3600\ s}{1\ hour}\right) = 9.4673\times10^{16}\ J$$

$$E = mc^2 \qquad \text{m} = \frac{\text{E}}{\text{c}^2} = \frac{9.4673\times10^{16}\ J}{\left(3.00\times10^8\ m/s\right)^2} = 1.05kg$$

Relativistic Velocity Addition

v_{BC} = velocity of train with respect to the ground.

v_{AB} = velocity of person with respect to the train.

v_{AC} = velocity of person with respect to the ground.

$$v_{AC} = \frac{v_{AB} + v_{BC}}{1 + \frac{v_{AB}v_{BC}}{c^2}}$$

When $v_{AB}v_{BC} << c^2$, $v_{AC} \approx v_{AB} + v_{BC}$

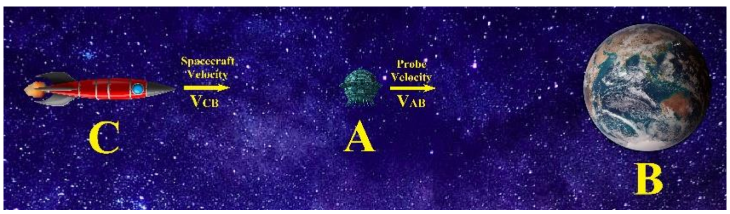
Once again Newtonian dynamics returns at low energy!

When $v_{AB} = c$, $v_{AC} = c$

The speed of light is the same for all observers!

To subtract, just make v_{AB} or v_{AC} negative

Example: A spacecraft approaching launches an exploration vehicle. An observer on Earth sees the spacecraft approach at 0.50c and the exploration vehicle approach at 0.70c. What is the speed of the exploration vehicle relative to the ship?



$$v_{AC} = \frac{v_{AB} + v_{BC}}{1 + \frac{v_{AB}v_{BC}}{c^2}} = \frac{0.70c - 0.50c}{1 + \frac{(0.70c)(-0.50c)}{c^2}}$$

$$v_{AC} = \frac{0.20c}{1 - 0.35} = \frac{0.20c}{0.65} = 0.31c$$

Note: $v_{BC} = -v_{CB}$. If the velocity of C with respect to B is positive, the velocity of B with respect to C is negative.

Distance

- In Galilean space the distance between two points is the same for all observers in inertial reference frames.

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$$

- In the Lorentzian space of special relativity, observers in different inertial frames agree on a different constant.

$$s = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (c\Delta t)^2}$$

- This is equivalent to treating a point as:

$$P = (x, y, z, ict)$$

- This would imply that space is equivalent to imaginary time and time is equivalent to imaginary space.