Part 7: Electromagnetic Waves and Optics

College Physics (Openstax): Chapters 24-27 Physics: Principles with Applications (Giancoli): Chapters 23-25

Maxwell's Equations

- $\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$ {Gauss's Law (for \vec{E})}
 - $\rho = charge/volume$

•
$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

- $\nabla \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$ {"Divergence"}
- $\nabla \cdot \vec{B} = 0$ {Gauss's Law (for \vec{B})}
 - No magnetic monopoles!

•
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
 {Faraday's Law}
• $\nabla \times \vec{E} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\hat{i} + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\hat{j} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\hat{k}$ {"Curl"}

•
$$\nabla \times \vec{B} = \mu_0 \vec{J}$$
 {Ampere's Law}

- J = current/area
- Maxwell changed Ampere's Law (basing this on symmetry)
- $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$ {Ampere's Law with Maxwell's displacement current}
 - The units of $\varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ are A/m², which are the same units as \vec{J} .

Maxwell's Derivation

• Maxwell found an interesting result for free space ($\rho=0$ and J=0)

$$\nabla \cdot \vec{E} = 0 \qquad \nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

• Take the curl of Faraday's Law

•
$$\nabla \times \left(\nabla \times \vec{E} \right) = \nabla \times \left(-\frac{\partial \vec{B}}{\partial t} \right)$$

• Apply two vector calculus identities

•
$$\nabla \times (\nabla \times \vec{E}) = \nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E}$$

• $\nabla \times \left(-\frac{\partial \vec{B}}{\partial t} \right) = -\frac{\partial}{\partial t} (\nabla \times B)$

- $\nabla(\nabla \cdot \vec{E}) \nabla^2 \vec{E} = -\frac{\partial}{\partial t} (\nabla \times B)$
- Apply Gauss's Law (rhs) and Ampere's Law (lhs)

•
$$-\nabla^2 \vec{E} = -\frac{\partial}{\partial t} \left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

•
$$\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

• This the equation for a travelling wave.

•
$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.998 \times 10^8 \,\mathrm{m/s}$$

- At the time, no one knew what light was, but they had measured its speed...
- Light is an electromagnetic wave!

Electromagnetic Waves



- Changing E creates B while changing B creates E.
- E and B are both \perp to v \rightarrow Transverse Wave
- No Medium is required.
- All electromagnetic waves travel through a vacuum (empty space) at the same speed
 - $c = 3.00 \text{ x } 10^8 \text{ m/s} \approx 186,000 \text{ miles/second}$
 - $c = f\lambda$



Electromagnetic Spectrum

Why do we only see such a small part of the electromagnetic spectrum?

Absorption Spectrum of Water



As a travelling wave, the product of the frequency and wavelength must be the velocity: $c = f\lambda$

Example: Some of the rays produced by an x-ray machine have a wavelength of 2.10 nm. What is the frequency of these electromagnetic waves?

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 m/s}{2.10 \times 10^{-9} m} = 1.43 \times 10^{17} \text{Hz}$$

Example: The brightest star in the night sky is Sirius, which is at a distance of 8.30×10^{16} m. When we are looking at this star, how far back in time (in years) are we seeing it?

$$x = vt$$

$$t = \frac{x}{c} = \frac{8.30 \times 10^{16} m}{3.00 \times 10^8 m/s} \left(\frac{1 hr}{3600 s}\right) \left(\frac{1 day}{24 hrs}\right) \left(\frac{1 year}{365.25 days}\right) = 8.76 years$$

i.e. Sirius is 8.76 light-years away.

Power in EM Waves

- Electric Field Energy Density: $u_E = \frac{Electric Energy}{Volume} = \frac{1}{2}\varepsilon_0 E^2$
- Magnetic Field Energy Density: $u_B = \frac{Magnetic Energy}{Volume} = \frac{1}{2\mu_0}B^2$
- Energy Density of EM Wave: $u_E + u_B = \frac{1}{2}\varepsilon_0 E_{RMS}^2 + \frac{1}{2\mu_0}B_{RMS}^2 = \varepsilon_0 E_{RMS}^2 = \frac{1}{\mu_0}B_{RMS}^2$
 - As E and B are sinusoidal, RMS values must be used for energy/power calculations.
 - The energy carried by the E and B fields is the same: $\frac{1}{2}\varepsilon_0 E_{RMS}^2 = \frac{1}{2\mu_0}B_{RMS}^2$

•
$$\varepsilon_0 E_{RMS}^2 = \frac{1}{\mu_0} B_{RMS}^2$$
 $E_{RMS}^2 = \frac{1}{\varepsilon_0 \mu_0} B_{RMS}^2$ $E_{RMS} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} B_{RMS} = c B_{RMS}$ $E_0 = c B_0$

Intensity
$$S = \frac{Power}{Area} = \frac{P}{A} = \frac{E_{TOT}}{tA}$$

• $E_{TOT} = uV = uLA = uctA$
• $S = \frac{E_{TOT}}{tA} = \frac{uctA}{tA} = uc = c\varepsilon_0 E_{RMS}^2 = \frac{1}{\mu_0} cB_{RMS}^2$

Example: The peak value of the magnetic field in an electromagnetic wave is 3.30×10^{-6} T. What is the peak value of the electric field?

$$E_0 = cB_0 = (3.00 \times 10^8 \,\mathrm{m/s})(3.30 \times 10^{-6} \,\mathrm{T}) = 990 \,\mathrm{V/m}$$

Example: The microwave radiation left over from the big bang has an average energy density of 4.00×10^{-14} J/m³. What is the rms value of the electric field of this radiation?

$$u = \varepsilon_0 E_{RMS}^2 \rightarrow E_{RMS} = \sqrt{\frac{u}{\varepsilon_0}} = \sqrt{\frac{4.00 \times 10^{-14} J/m^3}{8.85 \times 10^{-12} C^3 / (N \cdot m^2)}} = 67.2 \text{ mN/C}$$

Optics

- "Wave fronts" = Surfaces of constant phase
- "Rays" = Radial lines emanating from a source (in the direction of velocity)
- Spherical Wave: Wave front = Sphere



• Plane Wave: Wave front = Plane



Reflection

- **Specular Reflection** (smooth surfaces): Parallel incident rays become parallel reflected rays.
- **Diffuse Reflection** (rough surfaces): Parallel incident rays do not become parallel reflected rays.



<u>Law of Reflection</u>: $\theta_i = \theta_r$ (Angle of incidence equals angle of reflection)



Example:



Plane Mirrors

- The Image is:
 - (1) Upright/not inverted
 - (2) Same size as the object
 - (3) As far behind the mirror as the object is in front
- Images are found by **ray tracing**.



Real and Virtual Images

- In a Virtual Image, rays emanate from the reflecting surface rather than from the image itself.
- In a **Real Image** rays emanate from the image itself rather than from the reflecting surface.

Spherical Mirrors

- Spherical mirrors are parts of a sphere with one surface mirrored.
- A concave mirror is one where the inner surface is mirrored.
- A **convex mirror** is one where the outer surface is mirrored.
- The **principal axis** is the line passing through the sphere's center (C) and the midpoint of the mirror.
- A **paraxial ray** is near to and parallel to the principal axis.
- The focal point (F) is the point where paraxial rays converge after striking the mirror.
- The **focal length** is the distance from the focal point to the midpoint of the mirror, which is half the radius of the sphere.
- Rays not near to the principal axis do not converge at the focal point, resulting in blurred images known as **spherical aberration**.
- **Parabolic mirrors** are free from spherical aberration, but very costly to make.

Image Formation for Concave Mirrors

- Trace 3 rays from an object point which should all converge to an image point.
 - 1) Paraxial ray reflecting back through focal point (F).
 - 2) Directly through focal point (F), reflecting back as paraxial ray.
 - 3) Through center of curvature (C), reflecting back on itself.



Image is (1) inverted, (2) smaller than object, (3) real, and (4) between C and F.

This diagram also serves to cover the case where the object is between C and F. Swap the labels "Image" and "Object" and reverse the directions of the rays.

Concave Mirror Images

- If Object is **outside of C**, then the image is:
 - 1) Inverted (not upright)
 - 2) Smaller than the object
 - 3) Real (not virtual)
 - 4) Located between C and F
- If Object is **between C and F**, then the image is:
 - 1) Inverted (not upright)
 - 2) Larger than the object
 - 3) Real (not virtual)
 - 4) Located outside of C
- If Object is **inside F**, then the image is:
 - 1) Upright (not inverted)
 - 2) Larger than the object
 - 3) Virtual (not real)
 - 4) Located behind the mirror

Image Formation for Convex Mirrors

- Trace (the same) 3 rays from an object point which should all converge to an image point.
 - 1) Paraxial ray reflecting back away from focal point (F).
 - 2) Directly towards focal point (F), reflecting back as paraxial ray.
 - 3) Towards center of curvature (C), reflecting back on itself.



Image is (1) non-inverted, (2) smaller than object, (3) virtual, and (4) Inside F.

This diagram also serves to cover the case where the object is inside F for a concave mirror. Swap the labels "Image" and "Object" and reverse the directions of the rays.

Magnification (M) :
$$M = \frac{h_i}{h_o} \quad \left(\frac{height \ of \ image}{height \ of \ object}\right)$$

- Sign convention (mirrors)
 - 1) The height of inverted objects/images is negative
 - 2) Distances behind the mirror are negative
- d_o = Object Distance
- $d_i = Image Distance$

Note: "the image distance" and "the object distance" may be negative if behind the mirror, but the distance between the image and the mirror and the distance between the object and the mirror are always positive.



Two similar triangles: $\frac{d_i}{d_o} = \frac{-h_i}{h_o}$ (Note h_i is negative, so "- h_i " Is the positive distance)

• Magnification Equation: $M = -\frac{d_i}{d_o}$



Two similar triangles: $\frac{d_o - f}{f} = \frac{h_o}{-h_i} = \frac{d_o}{d_i}$ (Note: h_i is negative, so "- h_i " is the positive distance)

$$\frac{d_o - f}{f} = \frac{d_o}{d_i} \qquad \frac{d_o}{f} - 1 = \frac{d_o}{d_i} \qquad \frac{d_o}{f} = \frac{d_o}{d_i} + 1 \qquad \frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

• **Mirror Equation**: $\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$

Note: A positive value is usually given for the focal length of all spherical mirrors. By the sign convention the focal length of convex mirrors must be negative. Consequently you must add this negative sign where applicable. Example: A concave mirror has a focal length of 42.0 cm. The image formed by this mirror is 97.0 cm in front of the mirror. (a) What is the object distance? And (b) what is the magnification?

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \qquad \qquad \frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = \frac{d_i - f}{d_i f} \qquad \qquad d_o = \frac{d_i f}{d_i - f} = \frac{(+97.0cm)(+42.0cm)}{(+97.0cm) - (+42.0cm)} = 74.1cm$$
$$M = -\frac{d_i}{d_o} = -\frac{+97.0cm}{+74.1cm} = -1.31$$

Index of Refraction

- In free space (vacuum): $v_{Light} = c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3.00 \times 10^8 m/s$
- In other materials: $v_{Light} = \frac{1}{\sqrt{\epsilon\mu}} < c$
- **Index of refraction** (n): $n = \frac{C}{v_{Light}} \ge 1$

Example: A plate glass window (n=1.50) has a thickness of 4.00 mm. How long does it take light to pass perpendicularly through it?

$$n = \frac{c}{v_{Light}} \qquad v_{Light} = \frac{c}{n} = \frac{3.00 \times 10^8 \, m/s}{1.50} = 2.00 \times 10^8 \, m/s$$
$$x = vt \qquad t = \frac{x}{v} = \frac{4.00 \times 10^{-3} \, m}{2.00 \times 10^8 \, m/s} = 2.00 \times 10^{-11} \, s$$

Refraction

- Light rays bend when crossing an interface between different media. This is called **refraction**.
- When passing to a more dense medium $(n_2 > n_1)$, then the light will bend towards the normal.
- When passing to a less dense medium $(n_1 > n_2)$, then the light will bend away from the normal.



• Snell's Law: $n_1 Sin \theta_1 = n_2 Sin \theta_2$

Example: A light ray in air is incident on the surface of water at a 43.0° angle of incidence. Find (a) the angle of reflection, and (b) the angle of refraction. Note n_{air}=1.000 and n_{water}=1.333.

$$n_1 Sin \theta_1 = n_2 Sin \theta_2 \qquad \theta_2 = Sin^{-1} \left\{ \frac{n_1}{n_2} Sin \theta_1 \right\} = Sin^{-1} \left\{ \frac{1.000}{1.333} Sin 43.0 \right\} = 30.8^{\circ}$$

Total Internal Reflection

• If the angle of refraction is 90° or greater, there will be no refracted light (all is reflected).

$$n_1 Sin\theta_c = n_2 Sin90^\circ = n_2 \qquad \theta_c = Sin^{-1} \left(\frac{n_2}{n_1}\right) \qquad (n_1 > n_2)$$

- If $\theta_i \ge \theta_C$, then no light is refracted (all light is reflected).
- This is used in fiber optics where light reflects inside the fiber without escaping, allowing signals to be transmitted.

Example: One method of determining the refractive index of a transparent solid is to measure the critical angle when the solid is in air. If θ_c is found to be 40.5°, what is n for the solid?

$$n_1 Sin\theta_c = n_2$$
 $n_1 = \frac{n_2}{Sin\theta_c} = \frac{1.000}{Sin40.5^\circ} = 1.54$

Dispersion

- In many materials the index of refraction varies with frequency.
- When this happens light separates by frequency/color. This is called **dispersion**.
- Typically, violet and blue light scatter more (at greater angles) than red light.
- Rainbows are created when dispersion occurs as light passes through droplets of water.
- The sky is blue because violet and blue light scatter more (thus reaching your eyes from every possible angle).
- Sunrises/sunsets are orange and red because these colors don't scatter as much when going through the atmosphere for longer.

Lenses

- Rather than reflecting light to a focal point (as with mirrors), with lenses, light is bent to a focal point using refraction.
- Generally, we assume the object is on the left with the observer on the right.
- As with mirrors, image formation is found via ray tracing.



Image Formation: Converging Lenses

- Trace three rays from an object point, which should converge to an image point.
 - 1) Paraxial ray refracting through focal point (F) on far side of lens.
 - 2) Through focal point on near side, refracting as paraxial ray.
 - 3) Through center of lens without any refraction

Image is inverted and real.

Image Formation: Diverging Lenses

- Trace three rays from an object point, which should converge to an image point.
 - 1) Paraxial ray refracting away from focal point (F) on near side of lens.
 - 2) Towards focal point on far side, refracting as paraxial ray.
 - 3) Through center of lens without any refraction

Image is upright and virtual.

Equations and Sign Convention

- Trace three rays from an object point, which should converge to an image point.
- Magnification (M): $M = \frac{h_i}{h_o} \left(\frac{height \ of \ image}{height \ of \ object}\right)$
- Magnification Equation: $M = -\frac{d_i}{d_o}$
- Thin Lens Equation: $\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$
- Sign Convention:
 - <u>Focal Length</u>: + for converging lens / for diverging lens
 - <u>Object Distance</u>: + on left (real object) / on right (virtual object)
 - <u>Image Distance</u>: + on right (real image) / on left (virtual image)
 - <u>Magnification</u>: + if upright (same as object) / if inverted (opposite)
- **Example**: A diverging lens has a focal length of 32.0 cm. An object is placed 19.0 cm in front of this lens. Calculate (a) the image distance and (b) the magnification.

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \qquad \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{d_o - f}{d_o f}$$
$$d_i = \frac{d_o f}{d_o - f} = \frac{(+19.0cm)(-32.0cm)}{(+19.0cm) - (-32.0cm)} = -11.9cm$$
$$M = -\frac{d_i}{d_o} = -\frac{-11.9cm}{+19.0cm} = 0.627$$

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Lenses in Combination

- The image produced by the previous lens becomes the object of the next lens.
- **Example**: A converging lens has a focal length of 8.00 cm. An object is located 4.00 cm to the left of this lens. A second converging lens having the same focal length as the first is located 12.0 cm to the right of it. Relative to the second lens, where is the final image?

Start by finding the image of the first lens (ignoring the second).

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \qquad d_i = \frac{d_o f}{d_o - f} = \frac{(4.00 \text{ cm})(8.00 \text{ cm})}{(4.00 \text{ cm}) - (8.00 \text{ cm})} = -8.00 \text{ cm}$$

Then make a sketch to determine the position of this image relative to the second lens.

Interference

- Young's double slit experiment (1801)
- Interference is definitive proof that light is waves (not particles)
- Electric and magnetic fields obey superposition (they add)

• For interference to occur the light source must be monochromatic (single frequency) and coherent (unchanging phase relationship).

Double Slit Interference Young's double slit experiment (1801)

Example: A flat observation screen is placed a distance of 4.50 m from a pair of slits. The separation on the screen between the central bright fringe and the first order bright fringe is 37.0 mm. The light illuminating the slits has a wavelength of 490 nm. Determine the slit separation.

First order maxima $\rightarrow m=1$ As the small angle approximation applies (37.0 mm << 4.50 m), Sin $\theta \approx Tan \ \theta = Y/L$. $dSin \theta = \lambda$

$$d = \frac{\lambda}{\sin\theta} = \frac{\lambda L}{y} = \frac{(490 \times 10^{-9} \, m)(4.50m)}{(0.0370m)} = 59.6 \, \mu m$$

Single Slit Interference

- If the difference in path length between rays at the two edges is a full wavelength (or an integer multiple of a wavelength), then there are equal contributions from every phase, which cancel out (destructive).
- Destructive: $dSin\theta = m\lambda$, m = 0, 1, 2, 3, ...
- If the difference in path length between rays at the two edges is half a wavelength (or an integer plus ¹/₂ multiple of wavelengths), then there are equal contributions from only the first half of the sinusoid (constructive).
- Constructive: $dSin\theta = \left(m + \frac{1}{2}\right)\lambda$, m = 0, 1, 2, 3, ...
- The central maximum is wide as 0° remains a maximum.

Resolution

- **Resolving Power** is the ability to distinguish two nearby objects.
- For this we use a "single dot" (of radius D) instead of a "single slit".
- First dark band \rightarrow Sin $\theta = 1.22\lambda/D$
- $1.22 = (\text{First zero of Bessel Function})/\pi$
- **Rayleigh Criteria**: Two objects are just resolved when the first dark fringe of one object falls directly on the central bright fringe of the other.

• Separation that can be resolved:
$$\theta(rad) = 1.22 \frac{\lambda}{D}$$

- One additional effect must be accounted for!
 - When light goes from a less dense (smaller n) to a more dense (larger n) medium, it inverts (i.e. undergoes a 180° phase shift)
 - When light goes from a more dense (larger n) to a less dense (smaller n) medium, it doesn't invert (no phase shift)

•
$$\underbrace{2t}_{\substack{in \text{ path}\\length}} + \underbrace{\left\{\frac{1}{2}\lambda_{Film}\right\}}_{\substack{addedonly \text{ if}\\rays \text{ have differen}\\reflected phase shiff}} = \begin{cases} (m+1/2)\lambda_{Film} & Destructive\\ m\lambda_{Film} & Constructive \end{cases}$$

• t = thickness of film

•

Example: A thin layer of gasoline (n=1.40) sits on top of a puddle of water (n=1.33). It appears yellow as blue light (λ =469nm) is eliminated. Determine the minimum thickness of the film.

Minimum Thickness: m=1
$$t = \frac{\lambda_{Film}}{2} = \frac{335nm}{2} = 168nm$$

Bragg (X-Ray) Diffraction Diffraction off an atomic lattice

- D = Atomic Spacing
- Constructive interference occurs when the difference in path length $(2D \cdot Sin\theta)$ is an integer number of wavelengths $(m\lambda)$.
- Bragg's Law (constructive interference): Sin

$$\sin\theta = \frac{m\lambda}{2D}$$

- Interferometers are used to measure very small differences/changes in distance.
- If A=B, the light reaches the screen in phase (bright).
- If B-A= $\frac{1}{4}\lambda$, the light reaches the screen out of phase (dark). As the light travels 2A or 2B, the difference in path length would be 2B-2A= $\frac{1}{2}\lambda$.
- Changing the position of either mirror by $\frac{1}{4}\lambda$ will change from constructive to destructive and vice-versa.
- LIGO Laser Interferometer Gravitational wave Observatory (Livingston Louisiana)
 - LIGO is so sensitive that if it were measuring the distance between the Sun and Saturn it would detect changes equivalent to the width of a human hair.
 - Additional facilities in Washington state and Italy provide confirmation of signal.
- LISA Laser Interferometer Space Antenna
 - 3 satellites, 1 million kilometers apart, will create an interferometer in space
 - Expected launch date is now 2034

Polarization

• The polarization of light refers to the direction the electric field points.

