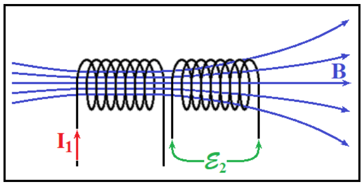
Part 6: AC Circuits

College Physics (Openstax): Chapter 23
Physics: Principles with Applications (Giancoli): Chapter 21

Mutual Inductance: $N_s\Phi_s = MI_P$



The magnetic field from the first (primary) coil will induce an emf in a nearby secondary coil.

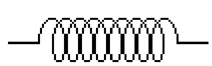
$N_s\Phi_s \propto B_P \propto I_P$

- This is not an equality as B through the secondary is not the same as the magnetic field through the primary (it falls off with distance)
- So we insert a constant and make an equality: $N_s\Phi_s = MI_P$
- $emf = -N_s\dfrac{\Delta\Phi_s}{\Delta t} = -\dfrac{\Delta(N_s\Phi_s)}{\Delta t} = -\dfrac{\Delta(MI_P)}{\Delta t} = -M\dfrac{\Delta I_P}{\Delta t}$
- Mutual Inductance (M) → Units 1 V·s/A = 1 H (one "Henry")

Example: The average emf induced in the secondary coil is 0.12V when the current in the primary changes from 3.4A to 1.6A in 0.14s. What is the average mutual inductance of the coils?

$$emf = -M\frac{\Delta I_P}{\Delta t} \quad \rightarrow \quad M = -\frac{emf \cdot \Delta t}{\Delta I_P} = -\frac{(0.12V)(0.14s)}{1.6A - 3.4A} = 9.3mH$$

Self Inductance:

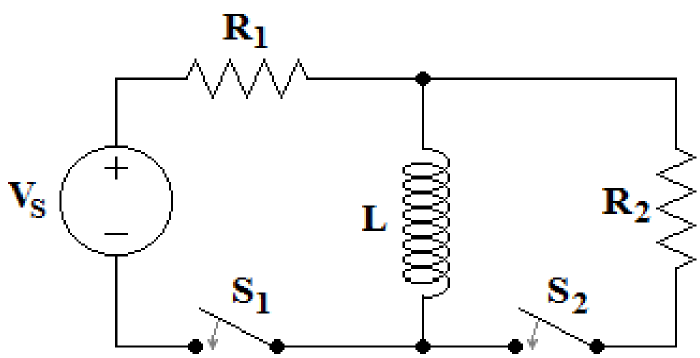
- Magnetic fields generated by individual loops create induced emfs in other loops in the coil. We call this "self inductance" or just "inductance".
- $emf = -M\dfrac{\Delta I_P}{\Delta t} \rightarrow emf = -L\dfrac{\Delta I}{\Delta t}$
- Inductance (L) → Units 1 V·s/A = 1 H (one "Henry")
- Schematic symbol for an inductor: 
- Energy Stored in an Inductor: $PE_L = \dfrac{1}{2}LI^2$
- Solenoid $L = \mu_0 n^2 Al$
  - $LI = N\Phi_B \rightarrow L = \dfrac{N\Phi_B}{I} = \dfrac{N(BA)}{I} = \dfrac{(nl)(\mu_0 nI)A}{I} = \mu_0 n^2 Al$
  - $PE_L = \dfrac{1}{2}LI^2 = \dfrac{1}{2}(\mu_0 n^2 Al)I^2 = \dfrac{1}{2\mu_0}(\mu_0^2 n^2 I^2)Al = \dfrac{1}{2\mu_0}B^2 Al$
- $Energy\ Density(of\ B) = \dfrac{Energy}{Volume} = \dfrac{1}{2\mu_0}B^2$

Example: A 2.00A current flows through a 20.0 cm long solenoid having 500 turns with each turn having an area of $3.00x10^{-2}m^2$. Determine (a) the inductance of the solenoid and (b) the energy stored in the solenoid.

$$L = \mu_0 n^2 A l = \left(4\pi \times 10^{-7} \frac{T \cdot m}{A}\right)\left(\frac{500}{0.200m}\right)^2 (3.00 \times 10^{-2} m^2)(0.200m) = 47.1mH$$

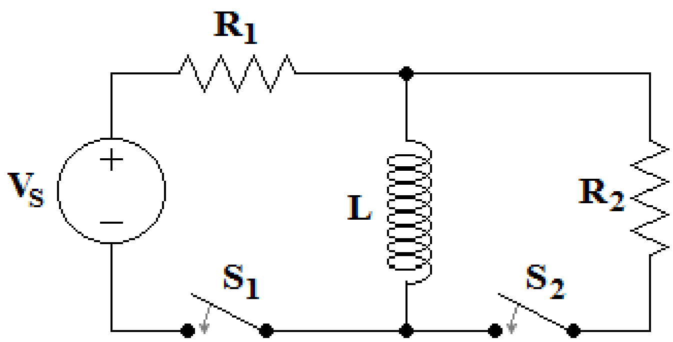
$$PE_L = \frac{1}{2} L I^2 = \frac{1}{2}(47.1239mH)(2.00A)^2 = 94.2mJ$$

RL Circuits Charging



- Assume the inductor L has no stored energy (i.e. no current) when the switch S_1 closes at t=0. S_2 remains open.
- As S_2 is open, no current flows through it. The right branch can be disregarded. We have a single loop with V_S, R_1 and L.
- As the inductor starts with no stored energy, this is referred to as "**charging**".
- The current through L builds gradually (the rate it grows is proportional to V_L). Thus, after the switch closes ($t=0^+$), I =0.
    - $I(t = 0^+) = 0 \qquad V_R = I(t = 0^+) R = 0 \qquad V_L(t = 0^+) = V_S$
- If we wait a long time ($t \rightarrow \infty$), the circuit will reach equilibrium.
    - $V_L = -L\frac{\Delta I}{\Delta t} = 0 \qquad V_S = V_R(t \rightarrow \infty) = IR_1 \qquad I(t \rightarrow \infty) = \frac{V_s}{R_1}$
- $V_S - R_1 I - L\frac{\Delta I}{\Delta t} = 0$
- $I = \frac{V_S}{R_1}\left\{1 - e^{\frac{-tR_1}{L}}\right\} \qquad V_L = V_S e^{\frac{-tR_1}{L}}$
- Time Constant : $\tau = L / R_1 \qquad V_L = V_S e^{\frac{-t}{\tau}}$

RL Circuits Discharging



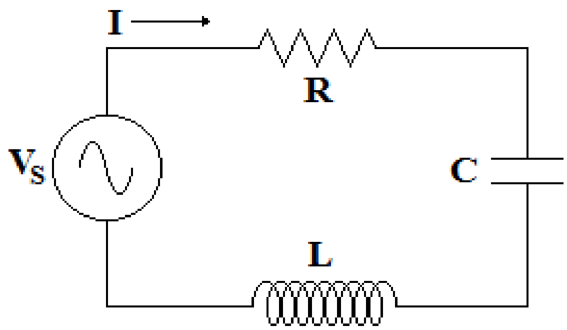
- The current through L is I_0 when the switch S_1 is opened and S_2 is closed at t=0.
- As S_1 is open, no current flows through it. The left branch can be disregarded. We have a single loop with R_2 and L.
- As the inductor starts with stored energy, this is referred to as "**discharging**".
- The current through L changes gradually (the rate it grows is proportional to V_L). Thus, after the switch closes ($t=0^+$), $I = I_0$.
  - $V_L = V_R(t = 0^+) = I_0 R_2$
- If we wait a long time ($t \rightarrow \infty$), the circuit will reach equilibrium.
  - $I(t \rightarrow \infty) = 0 \quad V_L(t \rightarrow \infty) = V_R(t \rightarrow \infty) = 0$
- Kirchoff's Voltage Law: $L\dfrac{\Delta I}{\Delta t} - R_2 I = 0$
- $V_L = I_0 R_2 e^{\frac{-tR_2}{L}} = V_0 e^{\frac{-tR_2}{L}} \qquad I = \dfrac{V_0}{R_2} e^{\frac{-tR_2}{L}} = I_0 e^{\frac{-tR_2}{L}}$
- Time Constant : $\tau = L / R_1 \qquad V_L = V_0 e^{\frac{-t}{\tau}}$

Example: A 200 mH inductor is connected in series with a 40.0 Ω resistor. If no energy is stored in the inductor when the pair are connected to a 12.0V battery (of negligible resistance), how long does it take until the current is 0.200A?

$$I = \frac{V_S}{R_1}\left\{1 - e^{\frac{-tR_1}{L}}\right\} \qquad \frac{IR_1}{V_S} = 1 - e^{\frac{-tR_1}{L}} \qquad e^{\frac{-tR_1}{L}} = 1 - \frac{IR_1}{V_S} \qquad \frac{-tR_1}{L} = \ln\left\{1 - \frac{IR_1}{V_S}\right\}$$

$$t = -\frac{L}{R_1}\ln\left\{1 - \frac{IR_1}{V_S}\right\} = -\frac{0.200\text{H}}{40.0\Omega}\ln\left\{1 - \frac{(0.200\text{A})(40.0\Omega)}{12.0\text{V}}\right\} = 5.49\text{ms}$$

AC Circuits



- As the same current flows through each element, let's start by assuming a simple AC current and determine the voltage across each element.
- $I = I_0 Cos(\omega t)$ $\quad\quad V_S = V_R + V_C + V_L$
- Resistor: $V_R = IR = I_0 R \cdot Cos(\omega t)$
- Inductor: $V_L = -\omega L I_0 Sin(\omega t) = \omega L I_0 Cos(\omega t + 90°)$
- Capacitor: $V_C = \dfrac{I_0}{\omega C} Sin(\omega t) = \dfrac{I_0}{\omega C} Cos(\omega t - 90°)$
- Define **Reactance**: $X_L = \omega L \quad X_C = \dfrac{1}{\omega C}$
- Ohm's Law holds for inductors and capacitors: $V_0 = I_0 X$

In AC circuits, inductors (L) and capacitors (C) act like frequency dependent resistors (X_L and X_C) with a 90° phase shift.

Example: A 200 mH inductor is connected in series with a 16.0 Hz source with a peak voltage of 20.0 V. (a) What current flows through the circuit? (b) At what frequency would the current be 2.00A?

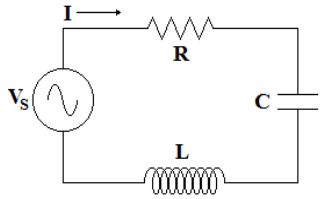
a) $X_L = \omega L = 2\pi f L = 2\pi(16.0Hz)(0.200H) = 20.1062\Omega$

$$I_0 = \frac{V_0}{X_L} = \frac{20.0\text{V}}{20.1\Omega} = 995\text{mA}$$

b) $X_L = \dfrac{V_0}{I_0} = \dfrac{20.0\text{V}}{2.00A} = 10.0\Omega$

$$f = \frac{X_L}{2\pi L} = \frac{10.0\Omega}{2\pi(0.200H)} = 7.96Hz$$

AC Circuits: RLC Circuit (Phasors)



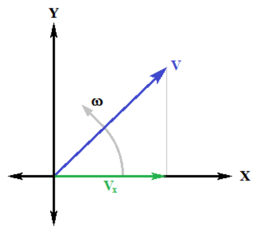
Resistor: $V_R = IR = I_0 R \cdot Cos(\omega t)$

Inductor: $V_L = I_0 X_L Cos(\omega t + 90°)$

Capacitor: $V_C = I_0 X_C Cos(\omega t - 90°)$

$V_S = V_R + V_C + V_L$

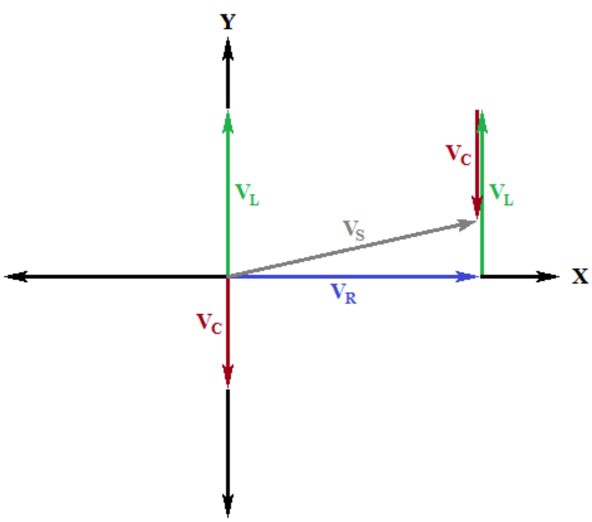
- We could add these using trig identities, but there's an easier way



If you take a vector of magnitude V and let it rotate at constant angular frequency (ω), the x-component of the vector will be: $V_X = V \cdot Cos(\theta) = V \cdot Cos(\omega t + \theta_0)$

We can represent V_R, V_L, and V_C as rotating vectors.

- As they rotate at the same angular frequency, their relative angular positions will remain constant. Thus, we can look at their orientation at any time (we'll choose t=0). Because the relative phase is all that matters, these are called "phasors".



$\vec{V}_S = \vec{V}_R + \vec{V}_C + \vec{V}_L$

The phase angle on V_R is 0°.

The phase angle on V_L is +90°.

The phase angle on V_C is -90°.

$V_{S-x} = V_R \qquad V_{S-y} = V_L - V_C$

$$V_{S0} = \left|\vec{V}_S\right| = \sqrt{V_{S-x}^2 + V_{S-y}^2} = \sqrt{V_R^2 + (V_L - V_C)^2} =$$

$$= \sqrt{(IR)^2 + (IX_L - IX_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$\theta_S = Tan^{-1}\left(\frac{V_L - V_C}{V_R}\right) = Tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$V_S = V_{S0} Cos(\omega t + \theta_S)$$

Complex Numbers: $z = a + bi$ where $i = \sqrt{-1}$

- **Complex Conjugate** $z^* = a - bi$
- Examples: $z_1 = 3 + 4i \qquad z_2 = 12 - 5i$
- Addition: $z_1 + z_2 = (3+4i) + (12-5i) = (3+12) + (4-5)i = 15 - i$
- Subtraction: $z_1 - z_2 = (3+4i) - (12-5i) = (3-12) + (4+5)i = -9 + 9i$
- Multiplication: $z_1 \times z_2 = (3+4i) \times (12-5i) = (3)(12) + (3)(-5i) + (4i)(12) + (4i)(-5i) =$
  $= 36 - 15i + 48i - 20i^2 = 36 + 33i + 20 = 56 + 33i$

- Division: $$\frac{z_1}{z_2}=\frac{(3+4i)}{(12-5i)}=\frac{(3+4i)}{(12-5i)}\cdot\frac{(12+5i)}{(12+5i)}=\frac{(3)(12)+(3)(5i)+(4i)(12)+(4i)(5i)}{(12)(12)+(12)(5i)+(-5i)(12)+(-5i)(5i)}=$$ $$=\frac{36+15i+48i-20}{144+60i-60i+25}=\frac{16+63i}{169}=\frac{16}{169}+\frac{63}{169}i=0.095+0.373i$$

<u>Euler's Theorem</u>: $e^{i\theta}=Cos\theta+iSin\theta$

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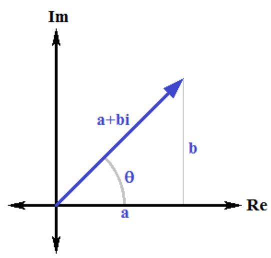

$$|z|=\sqrt{z\cdot z^*}=\sqrt{a^2+b^2} \qquad \theta=\mathrm{Tan}^{-1}\left(\frac{b}{a}\right)$$

$$a=|z|Cos\theta \qquad b=|z|Sin\theta$$

$$|z|e^{i\theta}=|z|Cos\theta+i|z|Sin\theta=a+bi$$

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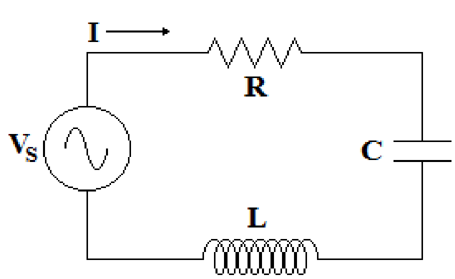
- Multiplication: $z_1\cdot z_2=|z_1|e^{i\theta_1}\cdot|z_2|e^{i\theta_2}=|z_1||z_2|e^{i(\theta_1+\theta_2)}$
- Division: $\frac{z_1}{z_2}=\frac{|z_1|e^{i\theta_1}}{|z_2|e^{i\theta_2}}=\frac{|z_1|}{|z_2|}e^{i(\theta_1-\theta_2)}$

<u>Examples</u>: $z_1=3+4i \qquad z_2=12-5i$

- Magnitudes: $|z_1|=\sqrt{3^2+4^2}=5 \qquad |z_2|=\sqrt{12^2+5^2}=13$
- Angles: $\theta_1=\mathrm{Tan}^{-1}\left(\frac{4}{3}\right)=53.1° \qquad \theta_2=\mathrm{Tan}^{-1}\left(\frac{-5}{12}\right)=-22.6°$
- Representation: $z_1=3+4i=5e^{i53.1°}=5\angle 53.1° \qquad z_2=12-5i=13e^{-i22.6°}=13\angle -22.6°$
- Multiplication: $z_1\cdot z_2=5e^{i53.1°}\cdot 13e^{-i22.6°}=5\cdot 13e^{i(53.1°-22.6°)}=65e^{i30.5°}$
  $65e^{i30.5°}=[65Cos(30.5°)]+i[65Sin(30.5°)]=56+33i$
- Division: $\frac{z_1}{z_2}=\frac{5e^{i53.1°}}{13e^{-i22.6°}}=\frac{5}{13}e^{i(53.1°+22.6°)}=\frac{5}{13}e^{i75.7°}=0.095+0.373i$

<u>AC Circuits: RLC Circuit</u> (Complex Numbers)



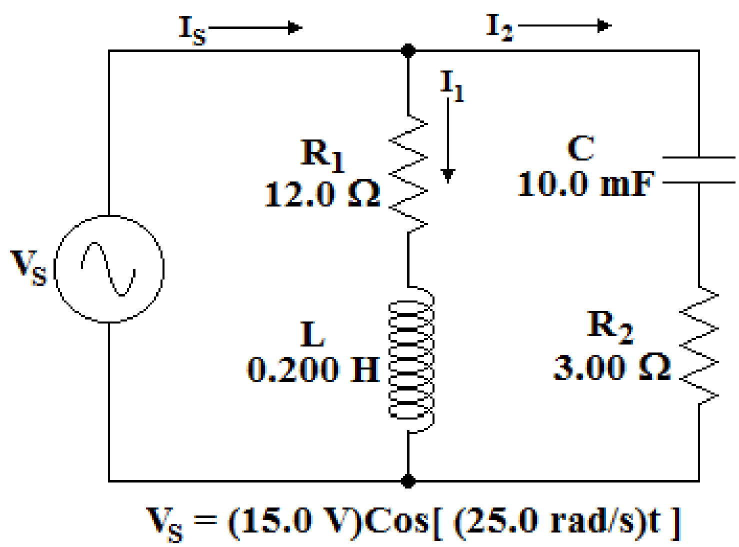
- Resistor: $V_R=IR=I_0R\cdot Cos(\omega t)$
- Inductor: $V_L=I_0X_LCos(\omega t+90°)$
- Capacitor: $V_C=I_0X_CCos(\omega t-90°)$
- Inductor: $X_L\to iX_L$ ("i" = +90°)
- Capacitor: $X_C\to -iX_C$ ("-i" = -90°)

- Real → "Resistance" (R)
- Imaginary → "Reactance" (X)
- Complex → "Impedance" (Z)

$$Z_{EQ} = R + iX_L - iX_C = R + i(X_L - X_C) \qquad |Z_{EQ}| = \sqrt{R^2 + (X_L - X_C)^2} \qquad \theta = Tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$I = I_0 Cos(\omega t) = I_0\angle 0° \qquad V = IZ = (I_0\angle 0°)(|Z_{EQ}|\angle\theta) = I_0|Z_{EQ}|\angle\theta$$

Example: Find I_1, I_2, and I_S.



1. Find X_L and X_C

$$iX_L = i\omega L = i(25.0\ rad/s)(0.200\ H) = i5.00\Omega$$

$$-iX_C = \frac{-i}{\omega C} = \frac{-i}{(25.0\ rad/s)(0.0100F)} = -i4.00\Omega$$

2. Find Z_{EQ1} and Z_{EQ2} (in both forms)

$$Z_{EQ1} = R_1 + iX_L = 12.0\Omega + i5.00\Omega$$

$$Z_{EQ2} = R_2 - iX_C = 3.00\Omega - i4.00\Omega$$

$$|Z_{EQ1}| = \sqrt{R_1^2 + X_L^2} = \sqrt{(12.0\Omega)^2 + (5.00\Omega)^2} = 13.0\Omega$$

$$\theta_1 = Tan^{-1}\left(\frac{X_L}{R_1}\right) = Tan^{-1}\left(\frac{5.00\Omega}{12.0\Omega}\right) = 22.62°$$

$$|Z_{EQ2}| = \sqrt{R_2^2 + X_C^2} = \sqrt{(3.00\Omega)^2 + (4.00\Omega)^2} = 5.00\Omega$$

$$\theta_2 = Tan^{-1}\left(\frac{-X_C}{R_2}\right) = Tan^{-1}\left(\frac{-4.00\Omega}{3.00\Omega}\right) = -53.13°$$

3. Use Ohm's Law to find I_1 and I_2 (Put them in both forms and write as a cosine)

$$I_1 = \frac{V_S}{Z_{EQ1}} = \frac{15.0V\angle 0°}{13.0\Omega\angle 22.62°} = 1.154A\angle -22.62°$$

$$I_1 = (1.154A)Cos(-22.62°) + i(1.154A)Sin(-22.62°) = 1.065A - i0.444A$$

$$I_1 = (1.15A)Cos[(25.0\, rad/s)t - 22.6°]$$

$$I_2 = \frac{V_S}{Z_{EQ2}} = \frac{15.0V\angle 0°}{5.00\Omega\angle -53.13°} = 3.00A\angle 53.13°$$

$$I_2 = (3.00A)Cos(53.13°) + i(3.00A)Sin(53.13°) = 1.80A + i2.40A$$

$$I_2 = (3.00A)Cos[(25.0\, rad/s)t + 53.1°]$$

4. Use Kirchoff's Current Law to find I_S (Put it in both forms and write as a cosine)

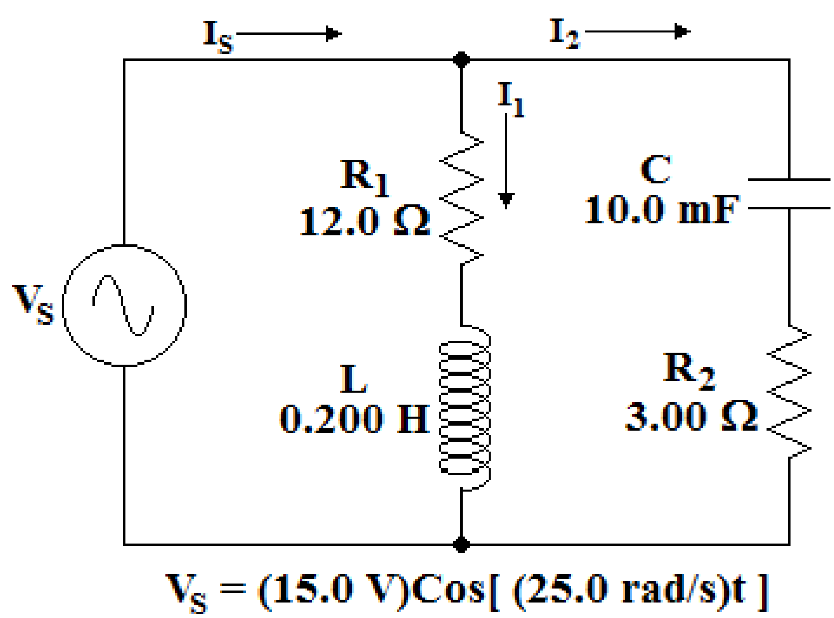
$$I_S = I_1 + I_2 = (1.065A - i0.444A) + (1.80A + i2.40A) = 2.865A + i1.956A$$

$$|I_S| = \sqrt{(2.865A)^2 + (1.956A)^2} = 3.469A \qquad \theta_S = Tan^{-1}\left(\frac{1.956A}{2.865A}\right) = 34.32°$$

$$I_S = 3.469\angle 34.32°$$

$$I_S = (3.47A)Cos[(25.0\, rad/s)t + 34.3°]$$

<u>Example</u>: Find Z_{EQ} and use it to get I_S.



1. Find X_L and X_C (From previous problem)

$$iX_L = i\omega L = i(25.0\ rad/s)(0.200\ H) = i5.00\Omega$$

$$-iX_C = \frac{-i}{\omega C} = \frac{-i}{(25.0\ rad/s)(0.0100F)} = -i4.00\Omega$$

$$Z_{EQ1} = R_1 + iX_L = 12.0\Omega + i5.00\Omega = 13.0\Omega\angle 22.62°$$

$$Z_{EQ2} = R_2 - iX_C = 3.00\Omega - i4.00\Omega = 5.00\Omega\angle -53.13°$$

2. Combine Z_{EQ1} and Z_{EQ2} like parallel resistors (put into both forms)

$$Z_{EQ} = Z_{EQ1} \parallel Z_{EQ2} = \frac{Z_{EQ1} \cdot Z_{EQ2}}{Z_{EQ1} + Z_{EQ2}} = \frac{(13.0\Omega\angle 22.62°)\cdot(5.00\Omega\angle -53.13°)}{(12.0\Omega + i5.00\Omega) + (3.00\Omega - i4.00\Omega)}$$

$$Z_{EQ} = \frac{65.0\Omega^2\angle -30.51°}{15.0\Omega + i1.00\Omega} = \frac{65.0\Omega^2\angle -30.51°}{15.03\Omega\angle 3.81°} = 4.324\Omega\angle -34.32° = 3.751\Omega - i2.438\Omega$$

$$15.0\Omega + i1.00\Omega = \sqrt{(15.0\Omega)^2 + (1.00\Omega)^2}\angle Tan^{-1}\left(\frac{1.00\Omega}{15.0\Omega}\right) = 15.03\Omega\angle 3.81°$$

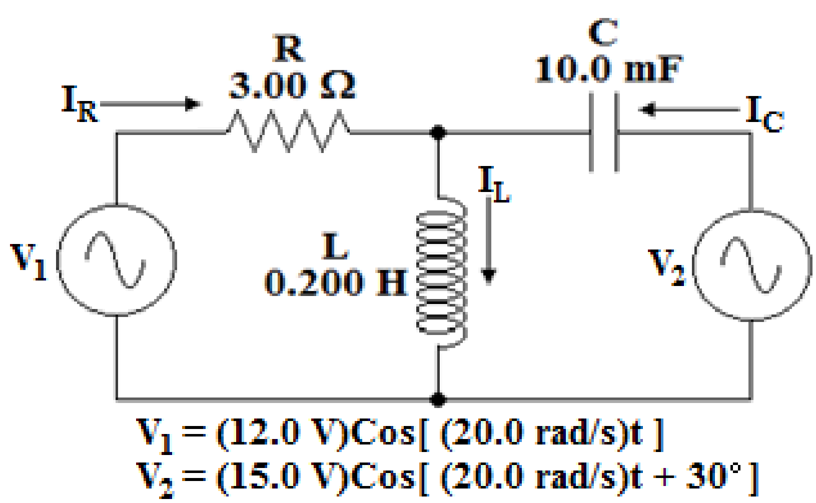
$$4.324\Omega \cdot Cos(-34.32°) + i4.324\Omega \cdot Sin(-34.32°) = 3.751\Omega - i2.438\Omega$$

3. Use Ohm's Law to find I_S.

$$I_S = \frac{V_s}{Z_{EQ}} = \frac{15.0V\angle 0°}{4.324\Omega\angle -34.32°} = 3.469A\angle 34.32°$$

$$I_S = (3.47A)Cos[(25.0\,rad/s)t + 34.3°]$$

Example: Find I_R, I_L, and I_C. (Let's use nodal analysis.)



1. Find X_L and X_C.

$$iX_L = i\omega L = i(20.0\ rad/s)(0.200H) = i4.00\Omega$$

$$-iX_c = \frac{-i}{\omega C} = \frac{-i}{(20.0\ rad/s)(0.0100F)} = -i5.00\Omega$$

2. Use Kirchoff's Current Law. $I_R + I_C = I_L$

3. Write Kirchoff's Law in terms of Node Voltages.

$$\frac{V_1 - V_A}{R} + \frac{V_2 - V_A}{-iX_C} = \frac{V_A}{iX_L}$$

$$\frac{12.0V\angle 0° - V_A}{3.00\Omega} + \frac{15.0V\angle 30° - V_A}{-i5.00\Omega} = \frac{V_A}{i4.00\Omega}$$

4. Solve for V_A.

$$\frac{1}{i} = \frac{1}{i}\cdot\frac{i}{i} = \frac{i}{i^2} = \frac{i}{-1} = -i \qquad \frac{12.0V\angle 0° - V_A}{3.00\Omega} + \frac{i(15.0V\angle 30° - V_A)}{5.00\Omega} = \frac{-iV_A}{4.00\Omega}$$

$$20(12.0V\angle 0° - V_A) + i12(15.0V\angle 30° - V_A) = -i15V_A$$

$$V_2 = 15.0V\angle 30° = (15.0V)Cos(30°) + i(15.0V)Sin(30°) = 13.0V + i7.50V$$

$$240V - 20V_A + i156V - 90V - i12V_A = -i15V_A$$

$$150V + i156V = 20V_A - i15V_A + i12V_A = (20 - i3)V_A$$

$$V_A = \frac{150V + i156V}{20 - i3} = \frac{216.4V\angle 46.12°}{20.224\angle -8.53°} = 10.70V\angle 54.65° = 6.19V + i8.727V$$

5. Use V_A to get the currents.

$$I_R = \frac{12.0V - (6.19V + i8.727V)}{3.00\Omega} = 1.937A - i2.909A = 3.4949A\angle -56.34°$$

$$I_C = \frac{13.0V + i7.50V - (6.19V + i8.727V)}{-i5.00\Omega} = \frac{6.81V - i1.227V}{-i5.00\Omega}$$

$$= 0.245A + i1.362A = 1.384A\angle 79.80°$$

$$I_L = \frac{6.19V + i8.727V}{i4.00\Omega} = 2.182A - i1.548A = 2.675A\angle -35.35°$$

6. Check your answers using Kirchoff's Current Law.

$$I_R + I_C = (1.937A - i2.909A) + (0.245A + i1.362A) = 2.182A - i1.547A = I_L$$

7. Write the currents as cosines if needed.

$$I_R = (3.49A)Cos[(20.0\, rad/s)t - 56.3°]$$

$$I_C = (1.38A)Cos[(20.0\, rad/s)t + 79.8°]$$

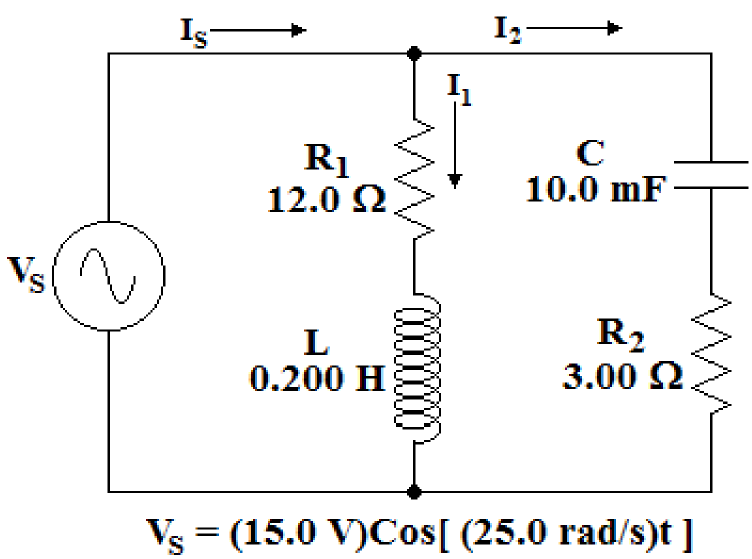
$$I_L = (2.68A)Cos[(20.0\, rad/s)t - 35.4°]$$

<u>AC Power</u>: $P_{AVG} = I_{RMS} V_{RMS} Cos(\theta_I - \theta_V)$

- The "Cos (Δθ)" is generally referred to as the "Power Factor".
- For a capacitor or inductor, the angle between its voltage and current is 90°, meaning these devices consume no power (they store and return it).

<u>Phase Matching</u>

- It is more efficient when the power factor for the source is 1. So, it is preferable that the source voltage and current are in phase. When they aren't an additional element (capacitor or inductor) may be added in series with the source to correct this.



$$Z_{EQ} = 4.324\Omega\angle -34.32° = 3.751\Omega - i2.438\Omega$$

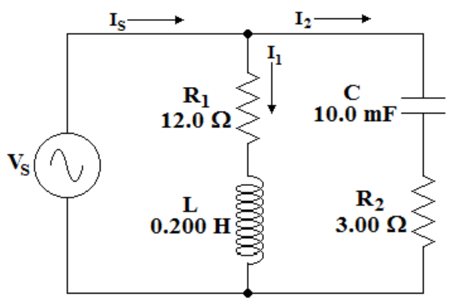
If we place "+i2.438Ω" in series, the imaginary part will cancel, making the power factor 1.

$$L = X_L / \omega = 2.438\Omega / 25.0 rad/s = 97.5mH$$

Frequency Limits

- High Frequency Limit (ω is very large): $\omega \to \infty$
  - Inductor: $X_L = \omega L \to \infty$ {Open Circuit}
  - Capacitor: $X_C = \dfrac{1}{\omega C} \to 0$ {Short Circuit}
- Low Frequency Limit (ω is very small): $\omega \to 0$
  - Inductor: $X_L = \omega L \to 0$ {Short Circuit}
  - Capacitor: $X_C = \dfrac{1}{\omega C} \to \infty$ {Open Circuit}

Example: Find I_S in the high and low freq limits.

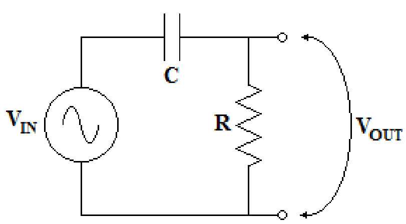


$V_S = (15.0\text{ V})\text{Cos}[\,(25.0\text{ rad/s})t\,]$

High Frequency: $I_S = \dfrac{V_S}{R_2} = \dfrac{15.0\text{V}}{3.00\Omega} = 5.00\text{A}$

Low Frequency: $I_S = \dfrac{V_S}{R_1} = \dfrac{15.0\text{V}}{12.0\Omega} = 1.25\text{A}$

Passive Filters



High Frequency: C → Short Circuit,
$V_{OUT} = V_{IN}$ (high frequency signals "Pass")
Low Frequency: C → Open Circuit,
$V_{OUT} = 0$ (Low frequency signals "cut off")

This is called a "High Pass Filter"

$$I = \frac{V_{IN}}{Z_{EQ}} \qquad Z_{EQ} = R - iX_c = R - \frac{i}{\omega C}$$

$$V_{OUT} = V_R = IR \qquad V_{OUT} = \frac{R}{Z_{EQ}} V_{IN}$$

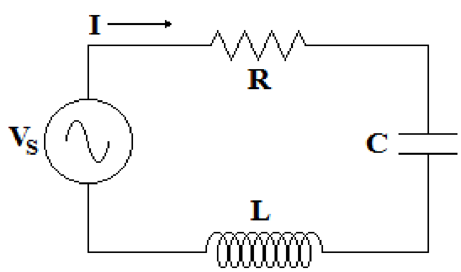
$$V_{OUT} = \frac{R}{R - \dfrac{i}{\omega C}} V_{IN} \qquad V_{OUT} = \frac{\omega RC}{\omega RC - i} V_{IN}$$

$$\text{Gain}(G) = \left|\frac{V_{OUT}}{V_{IN}}\right| = \left|\frac{\omega RC}{\omega RC - i}\right| = \frac{\omega RC}{\sqrt{(\omega RC)^2 + 1}}$$

The cutoff frequency (f_C) is the frequency where $G = \dfrac{G_{Max}}{\sqrt{2}}$

$$\omega_C RC = 1 \qquad \omega_C = \frac{1}{RC} \qquad f_C = \frac{\omega_C}{2\pi} = \frac{1}{2\pi RC}$$

<u>Resonance</u> : At a specific frequency, called the resonant frequency, the circuit will have a greater response (more current) than at others.

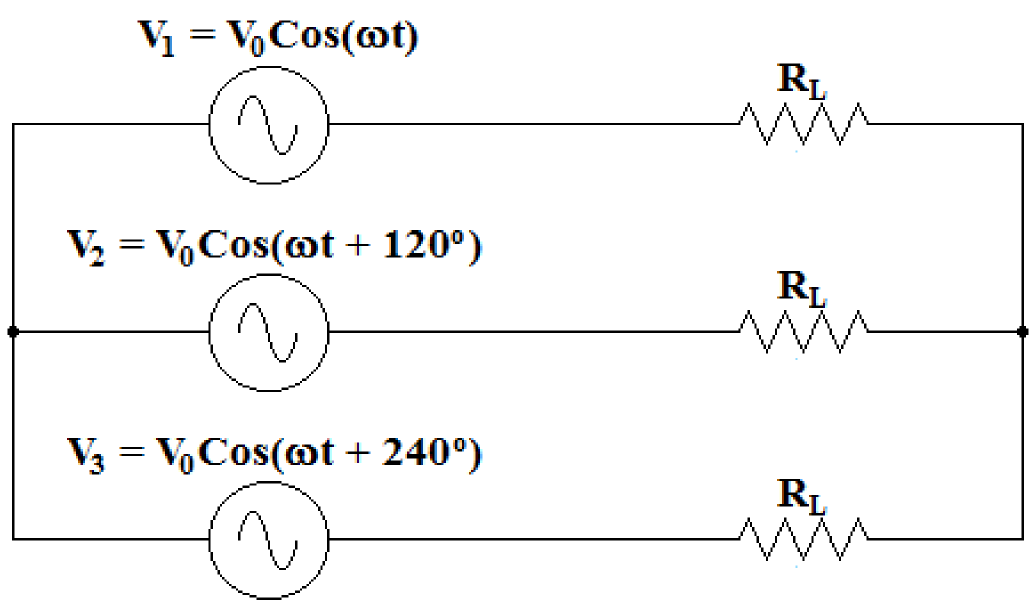


$$Z_{EQ} = R + iX_L - iX_C = R + i(X_L - X_C)$$

$$|I| = \left|\frac{V_S}{Z_{EQ}}\right| = \frac{|V_S|}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\omega_0 L - \frac{1}{\omega_0 C} = 0 \qquad \omega_0 = \frac{1}{\sqrt{\mathrm{LC}}}$$

<u>3-Phase Transmission</u>



By symmetry the connecting point on the right must be a ground (no ground wire needed!)