

## Part 3: Electric Potential (Voltage)

*College Physics (Openstax): Chapter 19*  
*Physics: Principles with Applications (Giancoli): Chapter 17*

### Electrical Potential :

$$\underbrace{\Delta V}_{\text{Change in electrical potential}} = \frac{\underbrace{\Delta PE}_q}{\underbrace{q}_{q=\text{test charge}}} = \frac{-W}{q} = \frac{-\vec{F} \cdot \vec{d}}{\underbrace{q}_{\text{Assuming F \& E are constant}}} = -\vec{E} \cdot \vec{d}$$

- Positive charges accelerate as they move from higher to lower V.
- Negative charges accelerate as they move from lower to higher V.
- Units: Volt (V)  $1V = 1 \frac{J}{C}$     Note:  $1 \frac{V}{m} = 1 \frac{J}{mC} = 1 \frac{N}{C}$

**Example:** Determine the number of particles (each with charge e) that pass between the terminals of a 12.0 V car battery when a 60.0 W headlight burns for an hour.

$$P = \frac{W}{t} \quad W = Pt = (60.0W)(3600s) = 216kJ$$

$$\Delta V = \frac{W}{q} \quad q = \frac{W}{\Delta V} = \frac{216kJ}{12.0V} = 18.0kC$$

$$N = \frac{q}{e} = \frac{18000C}{1.60 \times 10^{-19}C} = 1.13 \times 10^{23} \text{ particles}$$

**Example:** Determine the velocity of a proton, which after starting from rest is accelerated through a potential of 252 V.

$$|W| = qV = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2(1.60 \times 10^{-19}C)(252V)}{1.67 \times 10^{-27}kg}} = 220km/s$$

**The Electron Volt :** a new unit of energy

- An electron volt (eV) is the energy acquired by an electron moving through a potential of 1V
- $W = qV = eV = (1.60 \times 10^{-19}C)(1J/C) = 1.60 \times 10^{-19}J$
- It's normally used for atomic energy transitions.

**Example :** The Large Hadron Collider (LHC)

- In the LHC, protons are accelerated to an energy of 7TeV ( $7 \times 10^{12}$  eV).
- This is 7000 times the energy it takes to make a proton from nothing.
- This is equivalent to having the proton pass through a potential of  $7 \times 10^{12}$  V
- The proton beams of the LHC have the kinetic energy of 900 cars going 62 mph.

**V From E When E Is Constant** :  $\Delta V = -\vec{E} \cdot \vec{d}$

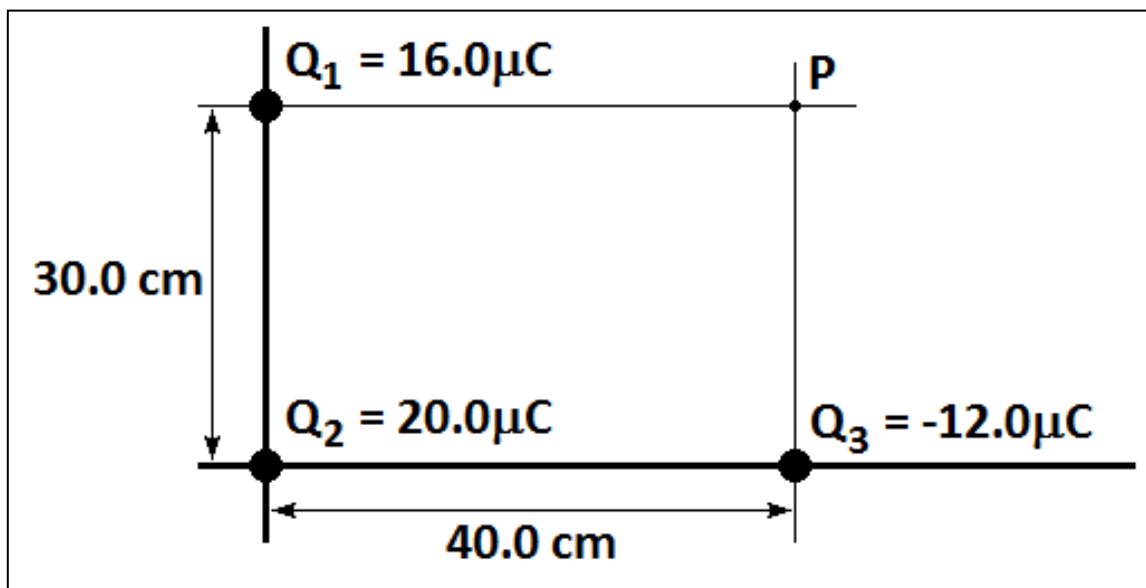
**Example**: What is the potential at the point (2.25 m, 1.50 m) measured with respect to the origin in a region with a uniform electric field  $E = (4.00 \text{ N/C})\hat{i} + (2.00 \text{ N/C})\hat{j}$  ?

$$\begin{aligned}\Delta V &= -\vec{E} \cdot \vec{d} = -\{(4.00 \text{ N/C})\hat{i} + (2.00 \text{ N/C})\hat{j}\} \cdot \{(2.25\text{m})\hat{i} + (1.50\text{m})\hat{j}\} \\ &= -\{(4.00 \text{ N/C})(2.25\text{m}) + (2.00 \text{ N/C})(1.50\text{m})\} = -12.0\text{V}\end{aligned}$$

**V For a Point Charge**:  $V_{\text{PointCharge}} = k \frac{Q}{r}$

- Using calculus, one can determine:  $\Delta V = k \frac{Q}{r_b} - k \frac{Q}{r_a}$
- To fix a reference, we set  $V=0$  at  $r \rightarrow \infty$
- $V$  is a scalar (we aren't finding magnitudes)  $\Rightarrow$  Keep the signs on the charges.
- Superposition: Just add up  $V$  for multiple charges/sources.

**Example**: Determine the electric potential at the point P due to the 3 point charges

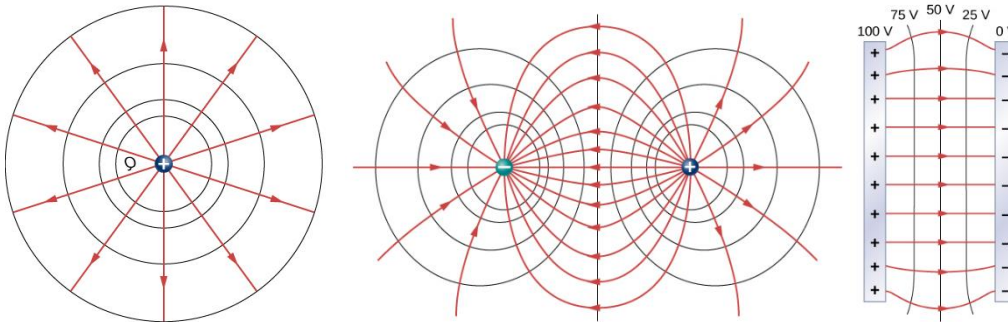
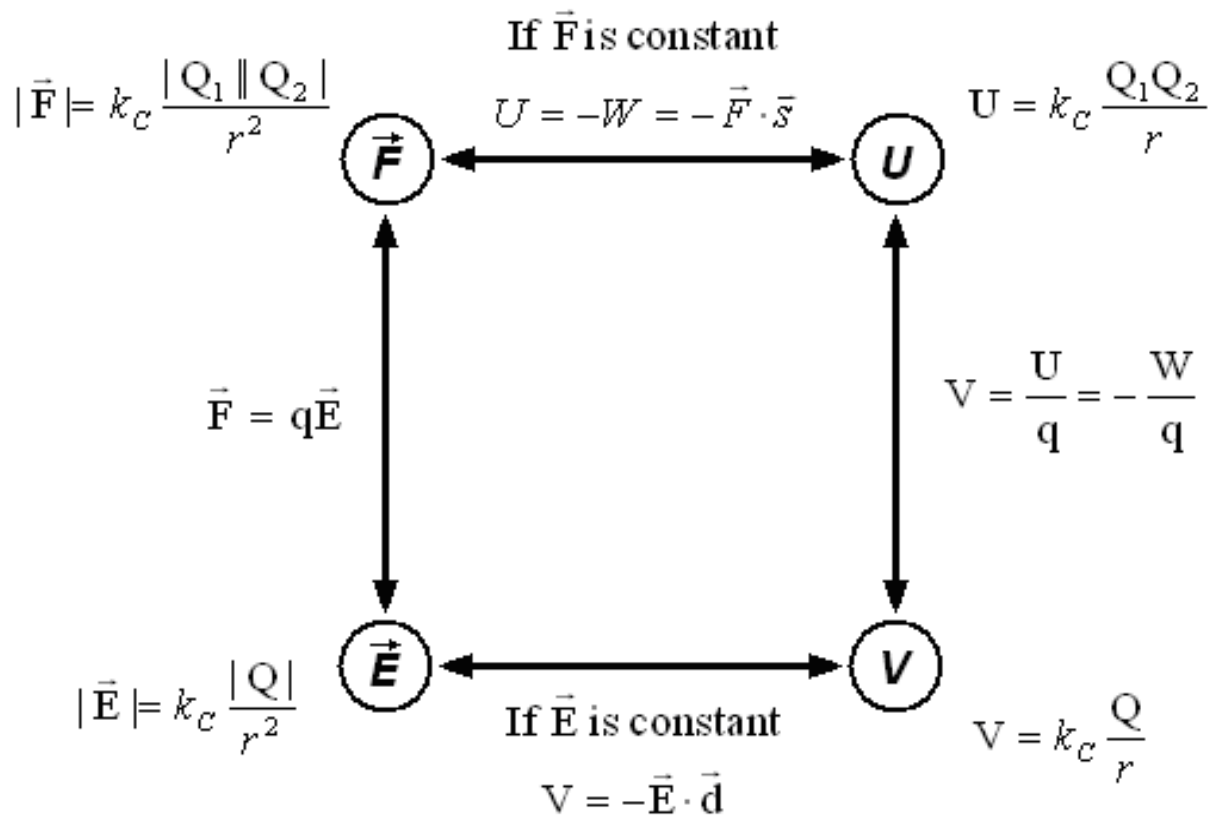


$$V_{TOT} = V_1 + V_2 + V_3 = k \frac{Q_1}{r_1} + k \frac{Q_2}{r_2} + k \frac{Q_3}{r_3} = k \left( \frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3} \right)$$

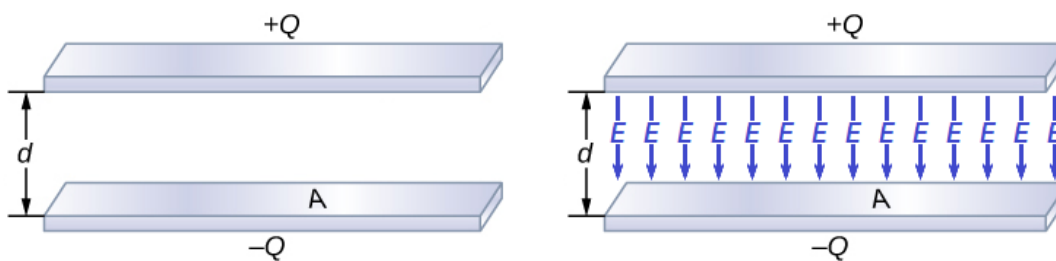
$$V_{TOT} = (9.00 \times 10^9 \text{ Nm}^2 / \text{C}^2) \left\{ \frac{16.0 \times 10^{-6} \text{ C}}{0.400 \text{ m}} + \frac{20.0 \times 10^{-6} \text{ C}}{0.500 \text{ m}} + \frac{-12.0 \times 10^{-6} \text{ C}}{0.300 \text{ m}} \right\} = 360 \text{ kV}$$

**Equipotential Surfaces** (Every point in the surface is at the same potential)

- No work is done by electrical forces when moving along an equipotential surface.
- The electric field is always  $\perp$  to the equipotential surface.

**Roadmap****Capacitance:**  $Q = CV$ 

- $Q$  = Charge
- $C$  = Capacitance
- $V$  = Electric Potential
- Units: Farad     $1 \text{ F} = 1 \text{ C/V}$



### Parallel Plate Capacitor

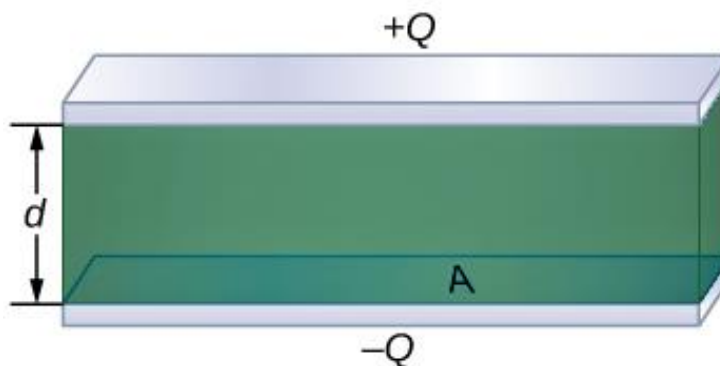
Electric field at surface of conductor:  $E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$

E is constant.

$$V = \vec{E} \cdot \vec{d} = Ed = \frac{Qd}{\epsilon_0 A} = \left( \frac{d}{\epsilon_0 A} \right) Q \quad Q = \left( \frac{\epsilon_0 A}{d} \right) V = CV \quad C = \frac{\epsilon_0 A}{d}$$

**Dielectrics:** A **dielectric** is an insulating material placed between the capacitor plates.

Let  $C_0$  be the capacitance without the dielectric, and  $C$  the capacitance with the dielectric.



$k$  = “dielectric constant”

$$\epsilon > \epsilon_0$$

$$\epsilon_0 \rightarrow \epsilon \quad \epsilon = \kappa \epsilon_0$$

$$\kappa > 1$$

$$C = \frac{\epsilon A}{d} = \frac{\kappa \epsilon_0 A}{d} = \kappa C_0$$

$$C > C_0$$

**Example:** What voltage is required to store  $7.20 \times 10^{-5} \text{ C}$  of charge on the plates of a  $6.00 \mu\text{F}$  capacitor?

$$Q = CV \quad V = \frac{Q}{C} = \frac{7.20 \times 10^{-5} \text{ C}}{6.00 \times 10^{-6} \text{ F}} = 12.0 \text{ V}$$

**Example:** A parallel plate capacitor has a capacitance of  $7.00 \mu\text{F}$  when filled with a dielectric. The area of each plate is  $1.50 \text{ m}^2$  and the separation between the plates is  $1.00 \times 10^{-5} \text{ m}$ . What is the dielectric constant of the dielectric material?

$$C = \frac{\kappa \epsilon_0 A}{d} \quad \kappa = \frac{dC}{\epsilon_0 A} = \frac{(1.00 \times 10^{-5} \text{ m})(7.00 \times 10^{-6} \text{ F})}{[8.85 \times 10^{-12} \text{ C}^2 / (\text{Nm}^2)](1.50 \text{ m}^2)} = 5.3$$

**Energy Storage in Capacitors:**  $PE = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

- $W = \sum_{\text{charges}} eV = V_{\text{avg}} \sum_{\text{charges}} e = QV_{\text{avg}} = \frac{1}{2}QV$
- $\frac{1}{2}QV = \frac{1}{2}(CV)V = \frac{1}{2}CV^2 \quad \frac{1}{2}QV = \frac{1}{2}Q\left(\frac{Q}{C}\right) = \frac{Q^2}{2C}$

### Energy Density of Electric Field

- Look at a parallel plate capacitor
- $PE = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\epsilon A}{d}\right)(Ed)^2 = \frac{1}{2}\epsilon AdE^2 = \frac{1}{2}\epsilon(Ad)E^2$
- $\text{Energy Density} = \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2}\epsilon E^2$

**Example:** A 6.00  $\mu\text{F}$  capacitor is charged to 12.0 V. (a) How much charge and energy are stored in the capacitor. (b) If the plates of the charged capacitor are then connected by a pair of thin conductors to the plates of a second, uncharged 12.0  $\mu\text{F}$  capacitor, what is the shared electrical potential once equilibrium has been achieved and how much energy is stored by the pair?

$$Q = CV = (6.00\mu\text{F})(12.0\text{V}) = 72.0\mu\text{C}$$

$$PE = \frac{1}{2}CV^2 = \frac{1}{2}(6.00\mu\text{F})(12.0\text{V})^2 = 432\mu\text{J}$$

$$V_1 = V_2 = V \quad Q_0 = Q_1 + Q_2$$

$$Q_0 = Q_1 + Q_2 = C_1V + C_2V = (C_1 + C_2)V$$

$$V = \frac{Q_0}{C_1 + C_2} = \frac{72.0\mu\text{C}}{(6.00\mu\text{F} + 12.0\mu\text{F})} = 4.00\text{V}$$

$$PE = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(6.00\mu\text{F} + 12.0\mu\text{F})(4.00\text{V})^2 = 144\mu\text{J}$$

**Example:** A 6.00  $\mu\text{F}$  capacitor is held at a constant electrical potential of 12.0 V. A dielectric ( $k = 3.00$ ) is then inserted between the capacitors plates. Assuming that friction is negligible, how much work must be done to insert the dielectric?

$$W_{\text{done}} = PE_{\text{final}} - PE_{\text{init}} = \frac{1}{2}C_FV^2 - \frac{1}{2}CV^2 = \frac{1}{2}(C_F - C)V^2$$

$$= \frac{1}{2}(\kappa C - C)V^2 = \frac{1}{2}(\kappa - 1)CV^2$$

$$= \frac{1}{2}(3.00 - 1)(6.00\mu\text{F})(12.0\text{V})^2 = 864\mu\text{J}$$

**Exercises**

1. An electron moves across a 5.00 V electric potential. Determine the magnitude of the electron's change in kinetic energy.
2. The plates of a parallel plate capacitor are separated by 1.00 mm. Determine the magnitude of the electric field between the plates when the voltage across the capacitor is 5.00 V.
3. A proton is headed directly towards an object with a charge that is 8350 times as large as the proton. When the proton is far away from the object its velocity is 1200 m/s. How close does it get to the object?
4. Two 25.0  $\mu\text{C}$  charges are fixed on the x-axis. One is placed at the origin, and the other is placed at  $x = 6.00$  m. Determine the electric potential at  $x = 3.00$  m on the x-axis.
5. Two charges are placed on an xy-plane. A 5.00  $\mu\text{C}$  charge ( $Q_1$ ) is fixed at the origin. A second charge ( $Q_2 = -10.0 \mu\text{C}$ ) is fixed at the point (7.00 m, 1.00 m). Determine the electric potential at the point (3.00 m, 4.00 m).
6. The plates in an air-filled parallel plate capacitor are squares with sides of length 1.00 cm separated by 88.5 nm. It is charged so that the voltage across it is 4.425 V. Determine (A) The capacitance of the capacitor, (B) The charge stored on the capacitor, (C) The magnitude of the electric field between the plates.
7. The capacitor from the previous problem is now filled with a dielectric ( $\kappa = 4.00$ ). Determine (A) The capacitance of the capacitor, (B) The charge stored on the capacitor, (C) The magnitude of the electric field between the plates.

**Exercise Solutions**

1. An electron moves across a 5.00 V electric potential. Determine the magnitude of the electron's change in kinetic energy.

$$|\Delta KE| = |W| = qV = (1.60 \times 10^{-19} \text{ C})(5.00 \text{ V}) = 8.00 \times 10^{-19} \text{ J}$$

2. The plates of a parallel plate capacitor are separated by 1.00 mm. Determine the magnitude of the electric field between the plates when the voltage across the capacitor is 5.00 V.

*Inside a parallel plate capacitor, E is constant.*  $|E| = \frac{V}{d} = \frac{5.00 \text{ V}}{1.00 \times 10^{-3} \text{ m}} = 5.00 \frac{\text{kV}}{\text{m}}$

3. A proton is headed directly towards an object with a charge that is 8350 times as large as the proton. When the proton is far away from the object its velocity is 1200 m/s. How close does it get to the object?

*When far away from a charge, the potential energy approaches zero. So all the energy is kinetic. At it's closest point to the charge, the proton will come to rest before being pushed away. When it comes to rest, it will only have electric potential energy.*

$$KE = \frac{1}{2}mv^2 = qV = eV = e\left(\frac{kQ}{r}\right) = e\left(\frac{8350ke}{r}\right) = \frac{8350ke^2}{r} = \frac{1}{2}mv^2$$

$$r = \frac{16700ke^2}{mv^2} = \frac{16700\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(1.60 \times 10^{-19} \text{ C})^2}{(1.67 \times 10^{-27} \text{ C})\left(1200 \frac{\text{m}}{\text{s}}\right)^2} = 16.0 \text{ m}$$

4. Two 25.0  $\mu\text{C}$  charges are fixed on the x-axis. One is placed at the origin, and the other is placed at  $x = 6.00 \text{ m}$ . Determine the electric potential at  $x = 3.00 \text{ m}$  on the x-axis.

$$V = V_1 + V_2 = \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2} = \frac{2kQ}{r} = \frac{2\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(25.0 \times 10^{-6} \text{ C})}{3.00 \text{ m}} = 150 \text{ kV}$$

*Note that  $Q = Q_1 = Q_2 = 25.0 \mu\text{C}$  and  $r = r_1 = r_2 = 3.00 \text{ m}$*

5. Two charges are placed on an xy-plane. A 5.00  $\mu\text{C}$  charge ( $Q_1$ ) is fixed at the origin. A second charge ( $Q_2 = -10.0 \mu\text{C}$ ) is fixed at the point (7.00 m, 1.00 m). Determine the electric potential at the point (3.00 m, 4.00 m).

$$r_1 = \sqrt{(3.00 \text{ m})^2 + (4.00 \text{ m})^2} = 5.00 \text{ m} \quad r_2 = \sqrt{(7.00 \text{ m} - 3.00 \text{ m})^2 + (1.00 \text{ m} - 4.00 \text{ m})^2} = 5.00 \text{ m}$$

$$V = V_1 + V_2 = \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2} = \frac{k}{r}(Q_1 + Q_2) = \frac{\left(9.00 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)}{5.00 \text{ m}}(5.00 \times 10^{-6} \text{ C} - 10.0 \times 10^{-6} \text{ C}) = -9.00 \text{ kV}$$

6. The plates in an air-filled parallel plate capacitor are squares with sides of length 1.00 cm separated by 88.5 nm. It is charged so that the voltage across it is 4.425 V. Determine (A) The

capacitance of the capacitor, (B) The charge stored on the capacitor, (C) The magnitude of the electric field between the plates.

$$A. C = \frac{\epsilon_0 A}{d} = \frac{\left(8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}\right)(0.0100 \text{ m})^2}{88.5 \times 10^{-9} \text{ m}} = 10.0 \text{ nF}$$

$$B. Q = CV = (10.0 \text{ nF})(4.425 \text{ V}) = 44.25 \text{ nC}$$

$$C. \text{ Inside a parallel plate capacitor, } E \text{ is constant. } |E| = \frac{V}{d} = \frac{4.425 \text{ V}}{88.5 \times 10^{-9} \text{ m}} = 50.0 \frac{MV}{m}$$

7. The capacitor from the previous problem is now filled with a dielectric ( $\kappa = 4.00$ ). Determine (A) The capacitance of the capacitor, (B) The charge stored on the capacitor, (C) The magnitude of the electric field between the plates.

$$A. C = \kappa C_0 = (4.00)(10.0 \text{ nF}) = 40.0 \text{ nF}$$

$$B. Q = CV = (40.0 \text{ nF})(4.425 \text{ V}) = 177 \text{ nC}$$

$$C. |E| = \frac{V}{d} = \frac{4.425 \text{ V}}{88.5 \times 10^{-9} \text{ m}} = 50.0 \frac{MV}{m}$$